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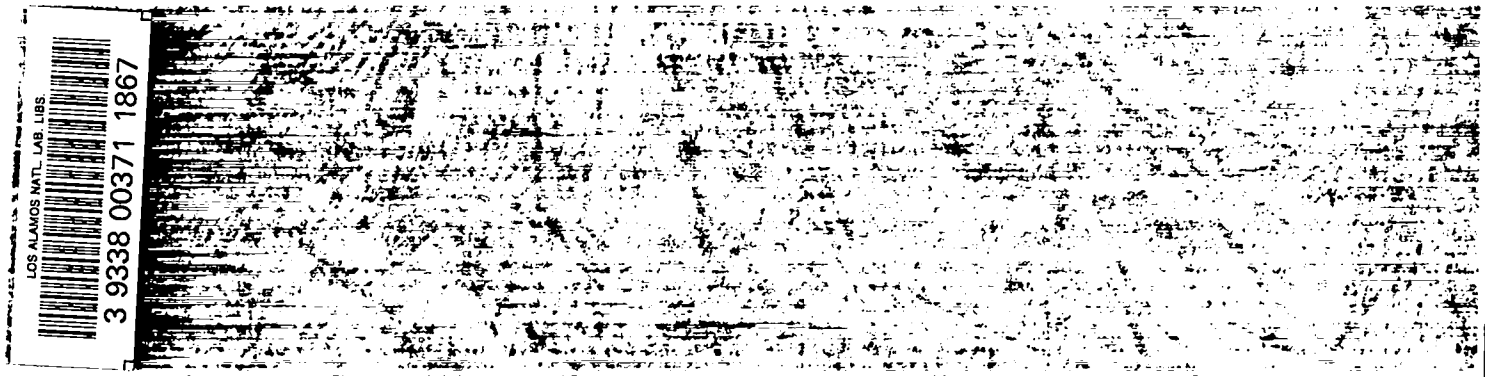
THE TWO-BODY t-MATRIX FOR A FREE-BOUND SYSTEM
PART I. APPLICATION TO NUCLEAR CHARGE EXCHANGE



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THE TWO-BODY t-MATRIX FOR A FREE-BOUND SYSTEM PART I. APPLICATION TO NUCLEAR CHARGE EXCHANGE

by

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Abstract

The charge exchange $X({}_{Z}A_{N,b-1}\,A_{N+1})Y$ can be reduced in a fully antisymmetrized description to one of elementary nature, $n+p \rightarrow n'+p'$. The t-matrix for the latter has, of course, to be averaged over the momenta distribution of the bound constituents. For sufficiently large bombarding energies and small Q values, the nucleon carried by the incident system may be considered as quasi-free. That carried by the outgoing system is then described in a similar fashion. This sort of approximation is consistent with the determination of t from an equation of the Bethe-Goldstone type. Solutions of this equation are sought which apply to the finite nucleus. Here, however, in contrast to the more usual situation one of the nuclear constituents is viewed as being in continuum states, the other in bound states.

The application of the development to $Be^{9}(He^{3},T)B^{9}*$ is solely for the purpose of concreteness. A major aspect of the analysis is that the virtual excited states of the intermediate system are taken into account. This is done by making specific assumptions concerning the single particle transitions brought about by the addition of a nucleon to the target nucleus. In addition, the possibility of a collective intermediate state excitation is considered. This is described through the introduction of a continuum two-particle bound state. Such a state gives

rise to the principal renormalization of the two-particle transition operator.

The final form of the elementary transition operator is one having a single-particle spectrum of excitations, renormalized by the coupling to the collective state. The transition operator is additive and can be classified both as to diagonality of its matrix elements and according to the resonance structure of these. In its form as a sum of diagonal and non-diagonal operators, the former is responsible for nuclear distortions, the latter for the physical change of state. It is the non-diagonal operator which determines the direct interaction processes. It, in particular, induces changes of state for just a few target constituents. The diagonal part of the interaction operator is moreover expressible as a sum of resonant and non-resonant terms. These are respectively contributions arising from the continuum bound state and those described as potential scattering. The latter again include virtual transitions of the interacting constituents.

Emphasis is put upon viewing the reaction problem in terms of the self-consistency requirements of the H-F method. This, together with procedures arising out of Brueckner theory, leads to the characterization of the interaction dynamics in the manner just described.

Introduction

We shall be dealing with the direct interaction process. In particular, we consider here nuclear charge exchange as described on such a picture. The charge exchange process is an interesting one since for low Q-values and large bombarding energies it is nearly an elastic process. It, on the other hand, can connect excited and ground state nuclear configurations. This means that it has a specific utility for the investigation of the properties of nuclear excited states.

In light nuclear systems where the role of the Coulomb force may be neglected, the isotopic spin may be considered a good quantum number. Thus in collisions involving such systems, we have a selection rule governing the transition which can occur. Rather than attempting to use the exchange reaction as a means of extracting information about the two-nucleon force, we assume this to be known. The force is considered to be charge-independent and its parameters given, for example, by the analysis of Gammel and Thaler (1).

What one does seek to do with the exchange reaction is to take into account the renormalizations of the two-body force. The concept of the direct interaction involves a statement that a few nucleons are involved in the reaction process. Additionally, or consequently, the energy sharing

between interacting systems ought to be slight. It is then consistent to argue that the renormalization of the two-body force is small. However, the simple picture is not altogether valid. We can get at the necessary modifications by considering those processes which give rise to a large shift of energy in the single quasi-particle states which define the self-consistent Hartree-Fock, (H-F), potential.

The modifications in the direct interaction picture have to do with the accounting for possible dynamical correlations which may occur in the two-nucleon system. In particular, the two-particle bound state of Brueckner, Eden and Francis (2) is an example of the correlated motion of which we speak. We shall see that the existence of this positive energy bound state has to do with the coupling of collective motions into the single quasi-particle spectrum. This special configuration, characterized by a large shift in energy for a single quasi-particle state, is not alone responsible for all of the couplings to collective excitations. There are other configurations which depend for their existence upon the couplings of three bodies, e.g., two particles and a hole. Some, not all, of these configurations are summed by the canonical transformation technique of Bogoliubov and Valatin (3). On the other hand, an alternative characterization of the coupling to the collective

motions is given by the Brueckner theory (4). Here one speaks of the direct excitation of a collective motion by a single quasi-particle. However, when both computations, that of the canonical method and that of the reaction-matrix method, are done self-consistently the condition emerging for the energy shift is a single one. There are to exist two quasi-particle states, one of which is an intermediate state. The difference in energy between these two states is to be approximately equal to the excitation energy for a given collective motion. (5,6) This remarkably simple result seems so intuitively obvious. It, on the other hand, is not a trivial statement. This is so for the reaction problem, at least, as long as we attempt to construct a self-consistent description in terms of the non-hermitian scattering matrix. Estimates made in a non-self-consistent way, and we shall always mean the self-consistency implied by the H-F method, (7) are unreliable, these being devoid of theoretical foundation. For the first thing, the force coupling a particle to a collective motion is not otherwise known, nor can it be estimated. The nature of the assumption concerning the long or short range character of the force presupposes the answer to the problem. Next, we know that the readjustments in the quasi-particle spectrum must not be so large as to imply correspondingly gross corrections in the ground state energy computations. The effects of the collective couplings are then to alter the

quasi-particle lifetimes, in the main. Lifetimes depend in a very delicate way upon the dynamical correlations. Here there is altogether no reason to trust other than self-consistent estimates.

We adopt the point of view that the reaction-matrix and canonical transformation methods are procedures whereby the program of successive diagonalizations in the H-F method is achieved. A unique answer exists in the overall scheme. Such being the case, one is entitled to take their common result, together with the more transparent physical statement, that already given, as a basis for understanding the two-particle bound state. In another language, this state is responsible for fluctuations about the neutron giant resonance.

A fully self-consistent program is difficult to carry out. Moreover, if we just wish to understand how the direct interaction model is to be corrected, it is equally unnecessary. The basic features which we hope will survive in an exact analysis are extracted from some naive models. Thus the discussion of Section II describes charge exchange as deriving from the interaction of two nucleons moving in a given potential well. One of the pair is in continuum states and the other in a bound state. An oscillator representation is used for the well. And, as we consider the reaction $B^9_\alpha(H^3_\alpha, T)B^{9*}$ for explicitness, only the states, those of

the p- and (s,d)-shell, of two major shells are taken into account. The relationship of the elementary two-nucleon charge exchange, proceeding through given virtual states, to the overall nuclear exchange is discussed in Section I.

To approximate the description of two nucleons moving in the self-consistent field of other nuclear constituents a many-particle formalism ought to be employed. Consequently, an equation of the Bethe-Goldstone type (8) is to be solved. It is also possible to arrive at an equally satisfactory description of the pair motion by classical techniques owing to reaction theory. We think here of the resonating group method as modified by certain elements from Watson's formulation of multiple scattering theory. (9) Such a composite representation can be put on a variational basis. This is enough to guarantee the required self-consistency of the representation. However, we do not carry out the program of systematic evaluation which is thereby implied. Nonetheless, it is this point of view which forms the basis of our entire discussion.

Apart from the single particle transitions to intermediate states, there may be others involving smaller energy denominators. These excitations may be described as collective. It is possible to include the contribution to the charge exchange made by virtual excitation of collective motions. This is done by carrying out a diagonalization of the two-nucleon force with respect to a given set of inter-

mediate states. Such is equivalent to the introduction of configuration mixing. The deduction of a collective state excitation is achieved by means of the specificity approximation to the internucleon force. Only T = 0 collective states are described by our analysis here.

Knowledge of the two-particle wave function is equivalent to that of a two-body transition operator. We shall refer to either of these according to convenience. It is possible to simply add the t-matrices for single particle and collective excitations. The result is an overall, or total, two-body operator describing, here, the charge exchange process. The underlying notion is that one contribution contains the scattering states of the two-particle spectrum of excitations. The remaining contribution represents the bound states of the spectrum. Such a simple procedure is not altogether correct. It, at the least, violates any sum rule which we might be able to construct for the two-particle strength function. Put more simply, part of the force considered to be available in the scattering states, has been lost in making up the bound states. This introduces the notion of renormalization. In correcting the simple addition of t-matrices, we are then led to this aspect in a natural way. Section III contains the relevant analysis.

In carrying out the renormalization discussion, certain additional features appear. The nuclear system to which a nucleon is added, thereby initiating the charge-exchange process, undergoes fluctuations of states. Accordingly,

that state in which we seek to add a particle may already be occupied. We account for the possibility of such processes by sketching the corresponding self-energy computation. The remainder of Section III is concerned with diverse topics departing somewhat from logical order. The computation of nucleon self-energies is achieved by means of a Green's function representation. This has the advantage of formal compactness. A procedure is considered for introducing nucleon-nucleus distortions in initial or final states, in an explicit way, which is just another way of expressing H-F self-consistency. A technique familiar from the formal theory of scattering is used to achieve this result.

There is to be found in Section IV a summary of our methods and some comparison of these with those familiar from earlier investigations. An enumeration of our approximations is also made, and as well that of the area of application of the analysis. Among the former is to be found: a statement of the ambiguity, here, of going off the energy shell; that relating to choice of a "chosen configuration"; and some consideration of t-matrix expansions as expressed by multiple scattering corrections.

I. Formal Preliminaries

The contents of this section have to do with the construction of an elementary two-body transition operator or t-matrix. From the latter it is possible, in a fully

antisymmetrized theory, to obtain the overall transition operator for a reaction between complex nuclear systems. We are restricted, by choice, to a discussion of nuclear charge exchange. In view of the fact that some advantage is offered by dealing with a concrete process, the reaction $Be^9(He^3,T)B^{9*}$ will serve to illustrate our procedures. Additionally, recent data, for 25 MeV He^3, by Wegner and Hall (10) require, we believe, all of the features treated here for its explanation. This data actually motivated the present analysis. However, the methods presented here have a more general validity. As such, they are not restricted to the specific reaction under discussion.

The exact transition amplitude for the given reaction is

$$\left\langle B_\mu^+ \, T_{1/2}^- \, \phi(\underline{k}_f ; \underline{R}_3) \, | \, v_0(S_B^-, S_T^+) \, | \, \Psi^{(+)}(B_{3/2}^- ; T_{1/2}^+) \right\rangle$$

We form, ignoring Coulomb forces, the objects $(He^3,T) = (T^+,T^-)$ and $(Be^9,B^9) = (B_\mu^-,B_\mu^+)$ characterized by a given value of the isotopic spin I; I = 1/2 in the ground state. In the space of isotopic spin there exists a 2 x 2 representation of the pairs of mirror nuclei involved in the reaction. The A = 9 isotopic spin multiplets form the basis of an irreducible representation of dimensionality four, corresponding to j = 3/2, in the space of total angular momentum, in their ground states. Otherwise, the quantum numbers for excited states must be characterized by $\mu = (\Pi_J, \epsilon_J, m_J)$ giving the energy, relative to the ground state, parity and angular momentum, J, $-J \leq m_J \leq J$ of a given state.

The A=3 multiplets have a ground state spin of 1/2. Standard notation has been used for the quantities appearing in the matrix elements: $\Psi^{(+)}$ is the stationary state of interaction, and has outgoing scattered wavesat infinity; v_0 is the interaction between the intial systems, having coordinates s_B^- and s_T^+; ϕ is the plane wave state describing, here, the relative motion of final systems, the c.m. momentum being $\underset{\sim}{k}_f$ and the relative coordinate being $\underset{\sim}{R}_3$.

The actual interaction between initial systems is replaced by a matrix F, using the statement

$$v_0 \Psi^{(+)} = F B^+_{3/2} T^+_{1/2} \phi(\underset{\sim}{k}_0 ; R'_3)$$

An integral equation can be derived for F in the usual way. We observe that F carries the instruction to antisymmetrize the product function upon which it acts. At this point, fractional parentage representations are introduced for the nuclear systems. The substance of these is contained in the statements

$$\omega_J^{M_J}(LTS) = {}^{2T+1,2S+1}L_J([\lambda])$$

$$= \sum \left\langle \ell^n TSL; [\lambda] \middle| \ell^{n-1} T'S'L'; [\lambda'] \ell \right\rangle \Psi(\ell; T'S'L'; [\lambda']),$$

$$C_J^{M_J} = \sum \alpha(LS)\, \omega_J^{M_J}(LTS).$$

An intermediate coupling representation is employed here. The totally antisymmetric wave functions for the A=3 and A=9 systems are represented by $C_J^{M_J}$. A variational principle

is used to determine the expansion coefficients α. Explicit details of the representation can be found in the papers of Jahn (11) and those of Levinson and Banergee (12). We represent the A=9 system as a $(1s)^4(1p)^5$ configuration. The closed core of s-state particles does not contribute to the spectrum of states. The representation for the A=3 system is a mixture of $(s^3),(s^2d)(d^2s)(p^2s)(p^2d)$ configurations, referred to a space-fixed axis.

To derive our matrix element, we observe that F symbolizes the interaction of the constituents of He^3 with Be^9. It is then a sum of terms. Only the $(1p)^5$ particles are imagined to participate in the direct interaction. The exchange of any two of these nucleons produces a change in sign of the matrix element. By assumption then $F = \sum(He^3;j)$ where $j = 1,\ldots,5$. The i-th matrix element in sum of matrix elements can be converted to the k-th. Upon making the exchange $(i \rightarrow k)$ in the i-th term, in the column vector F_i, we follow this by the same exchange in the row vector, which is just the final state amplitude. Now this amplitude is of indefinite symmetry with respect to the stated exchange. It then can be written as a sum of antisymmetric and symmetric terms in the pair (i,k). The latter has zero overlap with the column vector. Exchange of i and k in the antisymmetric term restores the overall sign and produces the matrix element of F_k from that F_i. We need only then to compute the matrix element of $F(He^3;j)$ and multiply this result by the factor of five.

The 3-body systems are treated somewhat differently. The fractional parentage representation still couples one particle to a core of A-1 particles. However, now, the core states of isotopic spin are restricted to the value T=0. Also only the core angular momentum L=0 is employed. The partition $\lambda = [21]$ in orbit will generate even parity, 3-body states with $L_{(3)} = 0,2$. The same eigenvalues may be produced by coupling the extra particle in states $\ell = 0,2$ to the core in the state L=0. In following this procedure, we pass to a cluster representation from that of the fractional parentage. The core will not be a deuteron, however, for it has a probability for being found in both singlet and triplet states. One further approximation is introduced and that is the core nucleons, much as those $(1s)^4$ in the A=9 system, do not induce transitions. This tells us how to count contributions to the overall transition amplitude. The 3-body wave functions will be given in an intermediate coupling representation.

The matrix element for the reaction can be written as

$$T_{0f} = 5 \sum \alpha^{+}_{\mu}(i)\,\beta^{-}(j)\,\alpha^{-}(i')\,\beta^{+}(j') \cdot \left\langle \omega_{\mu}(i; S_5^{+}; |p^5) \Pi^{1}_{1/2}(j; S_3^{-}) \phi(\underset{\sim}{k}_1; \underset{\sim}{R}_3) \middle| F_{01} \middle| \omega^{M'_J}_{J'}(i'; S_5^{-}; |p^5) \Pi^{h}_{1/2}(j'; S_3^{+}) \phi(\underset{\sim}{k}_0; \underset{\sim}{R}'_3) \right\rangle . \tag{1}$$

The four expansion coefficients $(\alpha^{\pm}, \beta^{\pm})$ have arguments which denote the various terms in the intermediate coupling expansions. For every nuclear state the appropriate coefficients are real and normalized to unity. We shall give the

explicit wave function decompositions involving these coefficients. The vector coupled eigenfunctions are labelled according to the sets of coordinates, e.g., $S_3^+ = (S_1, S_2, S_3)^+$, $S = (\underset{\sim}{r}, \underset{\sim}{\sigma}, \underset{\sim}{\tau})$, and the configurations, e.g., $1p^5$, insofar as is possible. The interaction operator F is interpreted as implying that the odd nucleon, "0", in the 3-body system has an interaction with the odd or vector coupled, nucleon, "1", in the 9-body system. Since we antisymmetrize in initial states, the nucleons can be given numerical labels. The antisymmetrization now occurs between nucleon "0" and those of the $1p^5$ configuration.

Now having written the matrix element out correctly, we introduce a further approximation. The exchange contributions implied by antisymmetrization are neglected. Despite the usual arguments having to do with A^{-1} corrections and the relative lack of importance of exchange for small momentum transfer, this is probably a bad approximation. It certainly destroys the self-consistency of the description. Moreover, in a proper many-particle formulation of the reaction, it would never be considered. We utilize the approximation here for expediency only. This is moreover a statement that the wrong mathematical apparatus has been employed. It is now possible to integrate out at this stage the core coordinates associated with both the A=3 and A=9 system. The necessary coordinate transformations in such a step are readily determined. We shall not discuss these here.

But for the appearance of the operator $F_{o\mu}$ which we subsequently denote as $t_{o\mu}$, the sense of the approximation is just that of plane wave Born theory. The various transformation coefficients, namely the c.f.p. and those for v.c., simply give formulas the appearance of complexity when written out explicitly. We shall indicate these in a symbolic way. For example, the coupling of one particle with quantum number q_1 coupled to a core, q_c , reproducing the quantum numbers q^- , exclusive of the total angular momentum, of the Be^9 ground state is symbolized by the transformation coefficient $B^-_{i'}; (M'_j q^-; q_c q_1)$. Similar use is made of the symbols $(\varkappa^+; \varkappa_0, \varkappa_2)$ in describing the couplings yielding the 3-body functions. With this new notation being included, the transition matrix element is given as

$$T_{of} = 5 \sum a^+_\mu(i) \beta^-(j) \alpha^-(i') \beta^+(j') B^+_i(\mu q^+; q_1 q_c)\, T^-_j(t\varkappa^-; \varkappa_0 \varkappa'_2) \cdot B^-_{i'}(M'_j q^-; q_0 q_c)\, T^+_{j'}(n\varkappa^+; \varkappa_1 \varkappa_2)\, (\varkappa'_2 q_0; \underline{k}_f \mid t_{01} \mid \varkappa_2 q_1; \underline{k}_0). \tag{2}$$

The basic two-body transformation matrix is fairly complicated. It carries, as did the original many-body matrix, the instruction to integrate over the variables, in the product space of position, spin, and i-spin, which are not physical observables. We shall not mention this further and it is a tacit assumption that our notation is consistent with regard to this aspect.

Some of the manipulations are clarified by giving the forms of the transformation matrix. Two equivalent representations are given below.

$$(F|t_{01}|0) = \int d\underset{\sim}{R}_{12}\, e^{i2/3\,\underset{\sim}{q}\cdot\underset{\sim}{R}_{12}} \Big\langle t^{-}_{\varkappa_2'}(\pi_1 ; \underset{\sim}{z}_1 - \underset{\sim}{R}_{12})\, F_{q_0}(S_0)\, e^{i\underset{\sim}{k}_f\cdot(8/9)\,\underset{\sim}{z}_1}\Big|$$

$$t_{01}\Big| h^{+}_{\varkappa_2}(\pi_0 ; \underset{\sim}{z}_0 - \underset{\sim}{R}_{12})\, F_{q_1}(s_1)\, e^{i\underset{\sim}{k}_0\cdot(1/3\,\underset{\sim}{z}_0 - 1/9\,\underset{\sim}{z}_1)}\Big\rangle \qquad (3)$$

$$(F|t_{01}|0) = \int \frac{d\underset{\sim}{\gamma}}{(2\pi)^3}\, G^{*}_{\varkappa_2'}(\underset{\sim}{\gamma})\, G_{\varkappa_2}(\underset{\sim}{\gamma} - 2/3\,\underset{\sim}{q})\Big\langle \psi(\pi_1)\, F_{q_0}(S_0)\, e^{i\underset{\sim}{\omega}_f\cdot\underset{\sim}{z}_1}\Big|$$

$$t_{01}\Big|\psi(\pi_0)\, F_{q_1}(s_1)\, e^{i\underset{\sim}{k}_0\cdot(\underset{\sim}{z}_1/27) + i\underset{\sim}{\omega}_0\cdot\underset{\sim}{z}_0}\Big\rangle \; ; \; \underset{\sim}{q} = \underset{\sim}{k}_0 - \underset{\sim}{k}_f \, , \qquad (4)$$

$$\underset{\sim}{\omega}_f = (8/27)\,\underset{\sim}{k}_f - \underset{\sim}{\gamma} \, , \; \underset{\sim}{\omega}_0 = (4/3)\,\underset{\sim}{k}_0 + \underset{\sim}{\gamma} - (2/3)\,\underset{\sim}{q} \, .$$

The additional notation $\pi \equiv (\underset{\sim}{\sigma}, \underset{\sim}{\tau})$ has been introduced for coordinates in the product space exclusive of position. All position coordinates are defined with respect to an origin at the c.m. of the $(1s)^4(1p)^4$ core of the 8-body system. The coordinate $\underset{\sim}{R}_{12}$ describes the motion of the c.m. of the neutron-proton core of the 3-body system. The remaining notation is altogether self-evident, being given that "0" refers to a proton and "1" to a neutron. Some additional comments are required here in order to abstract the physics from the mathematics. It should first be observed that the two-body matrix element is defined in a laboratory system. An average of the amplitude is performed either with respect to the coordinate or momentum distributions of initial and final (n-p) core systems of the 3-body nuclei. The appearance of the joint-probability distributions demonstrates that nuclear recoil is accounted for in the $A = 3$ systems. A "quasi-free" approximation would say that as a function of $\gamma = |\underset{\sim}{\gamma}|$, the momentum states in the vicinity of some γ_0 make the dominant contri-

bution to the average in Eq. (4). If $\gamma_0 << k_F, k_0$ is satisfied, the two-body amplitude having a slow variation, the dependence of the latter upon γ may be completely ignored. On the other hand, the joint-distribution in momentum is to be evaluated "in the vicinity of γ_0". It is very easy to make this notion quantitative. We do have at our disposal intermediate coupling representations of the 3-body wave functions. The corresponding single nucleon functions $G_{\alpha_i}(\pi_i ; \underset{\sim}{z}_i)$ are therefore known. The Fourier transforms of these functions may be computed for each possible nucleon state of orbital angular momentum. To each such distribution we may fit normalized Gaussians or combinations of these. The parameter of the functions characterize the spread of momentum about some average value. Products of momentum transforms may then be formed and the convolution implied by Eq. (4) carried out.

While our remarks have nearly the substance of the impulse approximation, they do carry some additional implications. These, as we shall see, have to do with the manner in which the two-body t-matrix is discussed. In this connection we note that the overall reaction amplitude cannot be described correctly by the formulas thus far given. One has neglected to take into account the very large contribution to the interaction between the initial $A = 3$ and $A = 9$ system. This contribution we shall describe as the Hartree-Fock (H-F) interaction, $\mathcal{V}_{H.F.}$. It is defined here as being diagonal in the states of the initial, $A = 9$ system.

The potential is constructed by placing the A = 3 nucleons in a given single particle state characterized by the relative momentum $\underset{\sim}{k}_0$ of initial systems. This state is then displaced to lower energy by an amount determined by the average binding per particle in the A = 3 initial nucleus. The potential $\mathcal{V}_{H.F.}$ is then computed as the sum of single particle H-F potentials between the A = 3 and A = 9 nucleons. We clearly then want to characterize the diagonal elements of $\mathcal{V}_{H.F.}$, computed for the state of the A = 3 system, as giving the optical potential for He^3 scattering from Be^9. The remaining interaction, namely $5xt_{0,}$, is a residual one and is considered non-diagonal in A = 3 and A = 9 states. The two-body t-matrix is taken then to satisfy an equation of the Bethe-Goldstone type. Its matrix elements are then computed between various H-F configurations. And, in accordance with the notion of Brueckner, the energy denominators appearing in the computation are always the excitation energies. These are measured away from the chosen configuration loosely described as giving the distorted-wave motion of He^3 relative to Be^9 in its ground state. Thus when we include the phase shifts for scattering of initial systems in the two-body t-matrix (this is done in one of the later sections), some of the self-consistency is built back into our description. By always then computing corrections to the H-F interaction, the incorrect description of residual interactions becomes of secondary importance.

II. Some Reaction Specifics

We continue our shell model formulation of the reaction process with a discussion of the nuclear wave functions. That for the $A = 9$ system is the more familiar and is examined first. French, Halbert and Pandya (13) have discussed the intermediate coupling shell model for Be^9. We slightly readjust the admixture parameters given by these authors. The ground and first excited state wave functions are here taken to be

$$(3/2^-) = 0.96\,{}^{2,2}P_{3/2}([411]) - 0.283\,{}^{2,2}D_{3/2}([411])$$

$$(1/2^-) = {}^{2,2}P_{1/2}([411]).$$

The arguments of the wave functions are the partition symbols $[\lambda]$. It is equally reliable to describe the same two states in B^9 by the same parameters. A comparison of the work cited with that of Kurath (14) indicates that this is not a completely empty approximation.

The $A = 3$ wave functions have been given by Young and Stein (15). The intermediate coupling assignments are rather more tentative here due to a failure to exploit a variational principle. Only one bound state exists in this case and for it we have

$$(1/2^+) = 0.697\ {}^2S'_{1/2} + 0.608\ {}^2S''_{1/2} + 0.384\ {}^4D_{1/2}.$$

The isotopic spin multiplicity of two is implied in the above and as well the partition symbol $[21]$. Both of the $A = 3$ nuclei are considered to be described by the pre-

ceding representation. Two S-states appear and these arise from the coupling of an S-state (n-p) system in singlet, S', and triplet, S", configurations to the odd nucleon. The D-state is generated from the triplet-odd-nucleon coupling and is probably 15% too large.

We shall not introduce any additional wave functions. However, states of nuclei other than those specifically mentioned are implied by our analysis. In fact we take as a starting point an assumption about the channels through which the reaction might proceed. Those which we have in mind are symbolized as follows:

$$Be^9 + He^3 \rightarrow d + B^{10*} \rightarrow d + n + B^{9*} \rightarrow T + B^{9*} \qquad (5a)$$

$$Be^9 + He^3 \rightarrow d + (n+p+Be^{8*}) \rightarrow T + B^{9*} \qquad (5b)$$

This is not to imply that we take these reaction schemes entirely literally. They will, however. serve as a guide for the manner in which virtual state transitions are incorporated into the analysis. The point of view is that the physical reaction occurs in a manner lying between two extreral descriptions. One of these is that given by the compound nucleus picture and the other that from direct interaction theory. There is no evidence that the reaction under study proceeds through a compound nucleus. In this and similar situations it is necessary to ascertain whether or not the simple momentum-transfer form factor of the direct interaction theory furnishes an adequate description. It is

characteristic of such form factors to drop off rapidly with increasingly large momentum transfers. Distorted wave theories, to be contrasted with plane-wave Born approximations, improve this situation somewhat, but not altogether. Antisymmetrization between the nucleons of target and projectile, as simulated, for example, in the theories of heavy particle stripping, is often required to control the form factor decay.

More to the point for our purposes is an alteration of the direction interaction theory so as to bring it closer to that of the compound nucleus. The inclusion of virtual transitions to intermediate states will accomplish this. The selection of a few important intermediate states is determined by the specifics of the reaction. In our example cited, we note that for excitations in excess of 8 MeV there are states in B^{10} which decay by neutron emission. The widths of the states are some 90 to 500 keV. The spins are not all identified but the isotopic spins seem to be T=1, the parities probably positive. In view of these remarks, the channel of (5a) is a likely one. There is also the possibility that the B^{10} spectrum is built in part on the motion of a neutron and proton couple to an excited Be^{8} core. The ground and first two excited states of Be^{8} are $T=0, J = 0^{+}, 2^{+}, 4^{+}$. The widths of the excited states are enormous, being, respectively, 1.20 and 6.7 MeV. The structure of the spectrum is reminiscent of that occurring in deformed

nuclei. In particular the ratio $(4^+-0^+):(2^+-0^+)$ of 4 is just about the value of 3.3 predicted by the theories of nuclear rotations. Although the states have the short lifetimes noted and decay by α-emission, we can describe them as collective. This is because in the p-shell there are only two parameters required to give level spacings. One sees this by looking at the partition [4] in orbit. In this sense either the phonon theory or that obtaining from an intermediate coupling expansion can be used to deduce the parameters. In including the channel (5b) it will be implied here that coupling occurs to 2.90 MeV state in Be^8, treated as a collective oscillation about the spherical shape.

In discussing the channel (5a) we seek to diagonalize the two-body interaction t_{oi}. This can only be done in part and produces a quantity $\langle t_{oi} \rangle$. Transitions are then induced by the operator $I_{oi} = t_{oi} - \langle t_{oi} \rangle$. The motivation for this is straight forward. We define as diagonal those operators not changing the state of the target nucleon. The operator I_{oi} is to change the states of both nucleons or that of "0" alone. Diagonalization then removes the H-F energy for the incident particle from the transition operator. If we work to what is identified as the second order in perturbation theory, then transitions of the nuclear particle must be taken into account. These we assume involve states in the p-shell, that partially occupied, and those in the (s,d)-shell. The inclusion of second order processes, analogous to self-energy insertions, makes it possible to account for

the effect of virtual single particle excitation upon the reaction. These will not in general be important as they involve large energy denominations. For example, the spin-flip transition $3/2^-$ to $1/2^-$ involves at least some 6 MeV of excitation, viewed as a single particle transition. On the other hand, the formalism is given in a general way to include situations when this conclusion cannot be reached.

It is convenient to use a harmonic oscillator basis for the discussion of the nuclear wave functions. For the A = 9 system, the oscillator spacing is $\hbar\omega$ = 19.7 MeV. The binding energy of the last neutron in Be^9 is 1.7 MeV and this defines the single particle well. There is a continuous spectrum of proton energies provided by incident He^3 system. For an incident energy of 25 MeV, the 18.75 MeV available in c.m. weights the proton spectrum heavily in this vicinity.(1) Approximately 5.6 MeV is required to separate the He^3 system into deuteron plus proton with zero energy of relative motion. In addition the average separation energy for He^3 is some 2.3 MeV. This value implies strong interaction between the three constituent nucleons. A small admixture of the D-state into the ground state wave function of He^3 is to be expected. On the basis of the cluster model used here, there is a small probability for obtaining protons of 1.9 MeV of binding. The proton spectrum may be characterized as extending over some 12 to 15 MeV, measured from the Be^9 ground state. The equivalent excitation energies measured from the Fermi

Note: Footnotes begin on page 85.

energy in Be^9 are obtained by adding 1.7 MeV to the values quoted. The imprecise qualitative statements are of course replaced by exact information contained in the nuclear form factors.

The two body wave[2] function, Ψ, in nucleons 0,1 is now assumed to satisfy an equation of the form given by Bethe and Goldstone. It is implicit that the potential v appearing here is that which generates the transition operator t_{01}. A chosen configuration Θ is determined carrying in principle the quantum labels for initial ground state systems. Our sole interest at this stage is to compute changes of state for the two nucleons previously singled out. All of the other nucleons in the A +3 system undergo no change of state. The function Θ may be thought of as the Slater determinant of wave functions $\eta(\underset{\sim}{k}_0; s_0)$, for particle "0" in state $\underset{\sim}{k}_0$, and $\phi(\mu_0; s_1)$, for particle "1" in state μ_0. A change of state for the two particles in question takes us to a new many-particle configuration. The latter has an excitation energy e with respect to the chosen configuration. It should be observed that while both nucleons move in the same well, that nucleon designated as "0" is in continuum states. The other nucleon, namely "1", is in bound states. We shall keep the two nucleons out of the occupied states in the chosen configuration. An operator, usually called Q, has the function of preserving the exclusion principle. In the Be^9, He^3 problem, Q must project off the occupied states of the p-shell. A form of the operator doing this is

$$Q = (1-\delta_{\alpha\alpha_c}) + \delta_{\alpha\alpha_c} Q(\alpha_c);$$

$$Q(\alpha_c) = 1 - [\ell_c]^{-1} \sum_{m_c=-\ell_c}^{\ell_c} \delta_{m_c m} . \qquad (6)$$

The operator acts in the product space of two nucleons, $Q = Q_0 Q_1$. If α_c refers to the quantum labels $(n\ell m)_c$ of states occupied in the chosen configuration, then $n_c = \ell_c = 1$, $[\ell_c] = 3$ for our problem. Eq. (6) indicates that 1/3 of the p-states are inaccessible to our interacting nucleons. We will often fail to introduce Q explicitly in the development to follow. It is understood that the factors of 2/3 are to be introduced where applicable.

The dynamical equation determining the two-nucleon motion is

$$\Psi = \Theta + \frac{Q}{\epsilon^+} v\Psi; \quad \epsilon^+ = e + i\eta . \qquad (7)$$

Introducing a complete set of states for a free-bound system, namely,

$$\Psi_{\mu\underset{\sim}{k}}(\underset{\sim}{r}_1 \underset{\sim}{r}_2) = \eta(\underset{\sim}{k};\underset{\sim}{r}_1)\, q_\mu(\underset{\sim}{r}_2),$$

we obtain a two-particle Green's function in configuration space,

$$(\underset{\sim}{r}_1' \underset{\sim}{r}_2' | G | \underset{\sim}{r}_1 \underset{\sim}{r}_2) = \sum\!\!\!\!\!\!\int_{\mu\underset{\sim}{k}} \frac{|\Psi_{\mu\underset{\sim}{k}}\rangle\langle\Psi_{\mu\underset{\sim}{k}}|}{E-\epsilon_k-\epsilon_\mu+i\eta}\ ; \ \epsilon_k = \hbar^2 k^2/2M$$

The unit operator appearing here should also be augmented by $1_s \times 1_I$, that for the space of charge-spin. It is simplest to think of the operators, e.g., that in spin, as decomposed in terms of the singlet and triplet projections. Then, for example,

$$1_s = P^{(1)} + P^{(2)} = 1/4\,(3 + \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2) + 1/4\,(1 - \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2)$$

and the four states (in spin) resolving the identity give rise to the operator representation. These familiar results are also implied in the formalism. The sums over spin and t-spin states are suppressed since they add nothing new in the way of details. One word about notation: the states $\eta\ (\underset{\sim}{k}_1 ; \underset{\sim}{r}_1)$ are those for scattering in the H-F potential. As such, they carry the real phase shifts arising from this potential. Moreover, the many-body aspect is thereby emphasized. The states $g_\mu(\underset{\sim}{r}_2)$ are those for bound particles in the H-F well. The outgoing radiation boundary condition, $\eta = \eta^{(+)}$ is implied for the scattering states. The generic notation (1,2) should not be confusing. We are still solving for the motion of an interacting system comprised of a bound and a free nucleon.

We generate a set of coupled linear equations starting from the statement

$$v\Psi = v\Theta + v\,\frac{Q}{\epsilon^+}\,v\Psi, \tag{7'}$$

and then operate from the left with $\psi_\alpha{}^*, \alpha = (\mu, \underset{\sim}{k})$ being a pair index. The equations have the form

$$\langle \psi_\alpha | v | \Psi \rangle = \langle \psi_\alpha | v | \Theta \rangle + \sum_\beta \int \langle \psi_\alpha | v | \psi_\beta \rangle \frac{1}{E - E_\beta + i\eta} \langle \psi_\beta | v | \Psi \rangle. \tag{8}$$

The wave function ψ_α should be a Slater determinant for the given state. In this way the exclusion principle

is taken into account in all matrix elements. A two-channel approximation to Eq. (8) is sought. Such a procedure was also employed in the Lamarsh and Fesbach (16) discussion of inelastic neutron scattering. To do the same thing here, we restrict the nucleon bound states in α and β to $\mu=\mu_0$ and $\mu=\mu_1$. If Ψ is expanded in terms of the ψ_α functions, a matrix of v connecting the two channels is encountered. One element of the matrix, v_{11}, describes excited state elastic scattering. The explicit appearance of this and the other elements is unavoidable. The explicit expansion of Ψ is circumvented through the "t-approximation",

$$v\psi = t\Theta \tag{9}$$

It is evident that $\mu=\mu_0$ is implied by Θ. The substance of the approximation (9) is that an affine transformation has been carried out upon the v-matrix. For this reason linear combinations of its elements determine a given element of t.

In the sum over intermediate states of Eq. (8), all processes leading to finite lifetimes are neglected. Two examples of these energy-conserving processes are shown in Fig. 1. The point here is that the inclusion of these introduces certain details. These are relevant to the physical problem. However, in a schematized version of the theory such as now presented, they are intrusive. We shall clarify any ambiguities of presentation in the following

section. For the present, we write down very symbolic formal statements. The processes which we can include, in addition to direct charge exchange, are shown in Fig. 2. These involve a change of state for one of the interacting particles. Bethe has argued that such processes are of order A^{-1} in large nuclei. They are described as the processes involving non-momentum conserving transfers. The processes are also small here owing to first, bad overlap and second, the large energy denominators. With these provisonal remarks being made, we write the coupled equations as

$$\left.\begin{aligned} b_{00} &= c_{00} + c_{01}\,\frac{1}{E-\epsilon_1}\,b_{10} \\ b_{10} &= c_{10} + c_{11}\,\frac{1}{E-\epsilon_1}\,b_{10} \end{aligned}\right\}. \qquad (10)$$

The two-particle energy ϵ_1 describes the target nucleon in (1d,2s) orbital with the incident nucleon still in ϵ_k^0 . An excitation of amount $\hbar\omega = 19.7$ MeV, the oscillator spacing, is required. The matrix element of the potential, that v_{11} , describes, in this unrealistic scheme, scattering of the incident nucleon from an initially excited target system. We may solve the second of Eqs. (10) as

$$b_{10} = \left(1 - c_{11}\,\frac{1}{E-\epsilon_1}\right)^{-1} c_{10} ,$$

whereupon,

$$b_{00} = c_{00} + c_{01}\,\frac{1}{E-\epsilon_1-c_{11}}\,c_{10} . \qquad (11)$$

Here, as in the preceding expression, the b's are the matrix elements of t and the c's those of v .

The expression (11) is suitable for a comparison with the two-body matrix element of (4), or nearly so. We shall indicate later how the two are related. At any rate if we were to completely ignore the role of Coulomb forces, (11) would be suitable for the description of the "elastic", ground-to-ground charge exchange. We mignt then argue that $b_{10} \approx c_{10}$ can be used to describe the ground-to-excited state exchange. This is untrue unless the excited state arises from a single particle transition. Thus the decomposition $t_{01} = \langle t_{01} \rangle + I$, $\langle t_{01} \rangle = b_{00}$ $(\epsilon_1 | I_{01} | \epsilon_1) = 0$, $(\mu_1 \epsilon_{k0} | I_{01} | \mu_0 \epsilon_{k0}) = b_{10}$ may not be altogether helpful. In particular, the first excited state in B^9 occurs at some 2.3 MeV of excitation. This is much too small to be accounted for by a single particle transition. The state in question is either $5/2^-$ or $3/2^-$. It can then be reached from the ground state by 1^+ magnetic (M1) or 2^+ electric (E2) transitions in both cases. The M1's are expected on the basis of a single particle model. The odd nucleon carrying all of the nuclear magnetic moment permits matrix elements of the moment to exist between ground and excited states. We further observe that in the fractional coupling model employed, the information about the excited A=9 states is carried by <u>geometrical</u> factors. This says that $\mathscr{R}_2' = \mathscr{R}_2$ in the form factor of Eq. (4). Alternately, the extra nucleon with $\ell = 1$ couples to L = 0,2,4 from [4] to form ground and excited states. We should then think of using (11) to compute both the Be^9 and Be^{9*} transitions.

The data of Wegner and Hall show that beyond 90^{o}, the c.m. cross section for ground and excited state exchange are practically identical. In the forward direction, charge exchange to the ground state shows a distinct peaking. The 10^{o}: 50^{o} cross section ratio is about 10. A similar comparison for the excited state yields a ratio of 3. The latter form factor is very nearly a constant function of momentum transfer. It is wholly unsafe to argue that one process represents a direct interaction and the other some sort of compound process.

We are stuck with the nuclear form factor. The geometrical coefficients will reduce overall ratio of excited:ground state yield. It is not likely that the 2.3 MeV change in Q will influence the dominant contribution from the nuclear form factor for small momenta transfer. And, certainly, the t-matrix of (11) is a slowly varying function of momentum transfer for small values of that quantity. A radical change in the t-matrix can be achieved by taking the exchange through another channel. That of (5b) will be considered here. In the intermediate states formed by adding the proton of He^{3} to the neutron of Be^{9}, excited states of B^{10} are encountered. The possible two particle states may be generated by operating on the B^{10} ground state as vacuum with two pairs of hole-particle creation and annihilation operators. Speaking now in j=j coupling, we form, symbolically, wave functions of the type

$$\Pi_\nu^J(1,2) = \sum_{i=1}^{2} C_i \left[\psi_{\ell m}(1)\, \tilde{\psi}_{\ell_1 m_1}(2) \right]_i^{J;M_J=0}$$

This basis with unperturbed energy $\epsilon_\nu = \sum_i^2 [\epsilon_i(p) - \epsilon_i(h)]$ (3) describes the motion of two hole-particle vibrations. Diagonalization of the short range interparticle force, e.g., one of zero range, in the basis produces a spectrum of states. Such states have a two-hole, two-particle character in terms of the associated shell model creation and annihilation operators. We may speak, however, of the excited states as arising from the presence of two quasi-particles. Charge exchange can now occur by the exchange of a vibration between two quasi-particles present. The diagrammatic process is indicated in Fig. 3. That figure implies that the charge-exchange force arises from the exchange of two-hole, two-particle vibrations. It is easy to show that the excitation spectrum of Be^{10} computed in this way is only approximately correct for the higher states. It is not literally suggested that pairing theory holds here. In fact, the older, seniority scheme of Racah (17) may be thought of as furnishing the basis of the discussion. No exact analogy holds with either formulation since we are interested in n-p interactions. These occur here in a region where both species are filling the same shell.

The energy denominators for two-particle states can be lowered from the 19.5 MeV previously quoted to about

10 MeV by the procedures first described. We can obtain something lower by treating the intermediate B^{10} system differently. This nucleus can be represented as a neutron and proton moving in the presence of an excited Be^{8} core. The excitations for the latter are loosely thought of as phonons "for quadrupole vibrations". In particular we consider the 1 phonon, 2^{+} state of the core. Such a state is also built from two-hole, two-particle excitations.(4) The vacuum here is the state with no phonon present. The physical state is then that with two "particles" and a single phonon present. The proton is added to the system at fairly high excitation. It does not have its motion altered to any appreciable extent. The neutron being at low excitation is influenced by the phonon. In another language, the neutron suppresses vacuum fluctuations. It has appreciable probability for being in the final states to which these fluctuations occur. As a result its self-energy is increased due to its coupling with the phonon.

A computation of the single-particle coupling to a phonon excitation has been done by Kisslinger and Sorensen (18). The interest there was in the influence of proton core oscillations upon the single neutron spectrum in the region of Ni. Their results are what one expects, with the "single particle" states shifted of the order of the phonon excitation energy. Our computation could be carried out in much the same manner by substituting an equivalent

repulsive interaction for the attractive one used previously. This statement is meant to emphasize the role played by the backward going graphs. The suppression of vacuum fluctuations is almost entirely due to these graphs. One such, occurring in second order, is shown in Fig. 4a.

It is necessary to deal with the two-particle spectrum in the present problem. The situation here is not so simple as that discussed in the previous paragraph. To discuss the intermediate state configuration of Fig. 4b, a device is employed. The mathematical artifice itself is attributed to Gottfried (19) and we re-interpret it for use here. The initial n-p system is uncorrelated, and apart from self-energy insertions, is not coupled to the 2^+ phonon. We account for this coupling while the two-nucleon system is in strong interaction. The intermediate states to which the nucleons scatter couple with the phonon for sufficiently small separation-in-energy, ω, of phonon and particle states. An estimate of ω is provided by observing that the $1p_-$ particle, some 4.6 MeV off the phonon energy, can effectively suppress the vibration. For purposes of enumeration, first, only the added proton is permitted a change of state. If then its energy lies within the interval of $0 \leq E_x \leq 7.5$ MeV, say, the phonon suppression is effective. Outside of this interval no coupling to the phonon occurs for the added nucleon. The condition is then imposed that both nucleons couple to the vibration. It is considered that the coupling

of just one nucleon does not affect the two-particle spectrum of excitations. Quite important is the fact that the transition of the target nucleon alone from $1p_{3/2}$ to $1p_{1/2}$ is also effective in suppressing vacuum fluctuations. This process has been represented graphically in Fig. 4a. The effectiveness of this last process depends critically upon the splitting $p_{3/2}-p_{1/2}$ in the single particle well. This we have estimated as 7.5 MeV from the neutron scattering on Be^9. The estimate was made by means of perturbation theory retaining terms up through second order.[5] The observed 10.7 Mev, T=1,π=+ state in B^{10}, whose spin we would **restrict** as $1 \leq J \leq 3$, is that dealt with here.

Equation (7') and (9) when taken together produce the integral equation

$$t = v + v \frac{Q}{\epsilon^+} t . \tag{12}$$

For definiteness and purposes of reference, the energy denominator is written explicitly as

$$\epsilon^+ = E(A+1) - \hbar(0) - \hbar(1) + i\eta . \tag{13}$$

The symbols h(j) refer to the H-F energy operator for the nucleons, e.g., h(j) is that for nucleon j. The energy of the chosen configuration is $E=E_0(A+1)$. However we are going to discuss now the spectral representation of t, namely t(E). This takes Eq. (12) out of its original context, at least for the moment. The identity is resolved

over H-F states in two-particles, and ϵ^+ of (13), is diagonal in such states, according to definition. An arbitrary matrix element of (12) is, in pair-index notation,

$$\langle \alpha | t | \beta \rangle = \langle \alpha | v | \beta \rangle + \sum\!\!\!\!\!\!\int_{\gamma} \frac{\langle \alpha | v | \gamma \rangle \langle \gamma | t | \beta \rangle}{E - \epsilon_\gamma + i\eta} . \tag{14}$$

The approximation assuming constant matrix elements of for certain states ϵ_ν, is introduced. This is formally the statement

$$\begin{aligned} \langle \alpha | v | \beta \rangle &= \text{const}, \ (E^* \leq \epsilon_\alpha \ \epsilon_\beta \leq E^* + \omega) \\ &= 0, \text{ otherwise.} \end{aligned} \tag{15}$$

The Gor'kov factorization in superconductivity utilizes the preceding statement. For the constant in our equation, we use

$$\langle \mu | v | \nu \rangle = \lambda \, u(\mu) u(\nu). \tag{16}$$

Here λ is an undetermined parameter giving both sign and magnitude of the interaction. It is also thereby implied that v is unknown. In particular then the v of Eq. (12) is certainly not that of Eq. (7'). To avoid confusion we should have employed a notation of $\tilde{v}$. Similarly, t of Eq. (12) is unknown and should be called $\tilde{t}$. It is possible to establish that $\tilde{v}$ is some part of v. The term residual force applies to $\tilde{v}$. In addition it can be seen from the Yamagouchi factorization of Eq. (16) that $\tilde{v}$ is what Mottelson (20) would describe as a "specificity force". Roughly speaking, such a force is that undiagonalized by the H-F

procedure. It nevertheless has the property of describing long range correlation in the nuclear medium. In a finite nucleus these correlations arise from the existence of collective degrees of freedom. It is possible to extract these motions from the cluster corrections of Brueckner theory. Such has already been done for the elastic scattering of neutrons. This procedure is described elsewhere.(4) The present method, Eqs. (12) - (16), gives the same answer as that of a more precise analysis. It also displays both of the approximations, one being specificity, which are required. The other, the adiabatic condition, has to do with the slow variation of matrix elements implied by (16). A word or so about the backward going graphs is necessary. These have only been taken into account through our numerical estimate of ω. This is certainly unfortunate and as well incorrect. It is otherwise impossible to reproduce the energy-weighted sum rules for distribution of multipole strengths in the nuclear excitation spectrum. (21), (22) The importance of ground state correlations, i.e., those already present in the medium, is the greater the closer one comes to zero frequency.

If now we write

$$\langle \mu | t | \nu \rangle = u(\nu) f(\mu)$$

and substitute this, together with (16), into (14), we shall obtain

$$f(\beta) = \frac{\lambda\, u(\beta)}{1 - \lambda I(E)} \; ; \quad I(E) = \sum \frac{u^2(\gamma)}{E - \epsilon_\gamma + i\eta} \; . \tag{17}$$

We define next a parameter λ' equal to $\langle u^2(\gamma)\rangle_{av}\, \lambda$, the average-in-energy yielding a mean square matrix element connecting initial state β to those $\{\gamma\}$. Then the condition for the existence of a state in the two particle spectrum of excitations is

$$1 - \lambda\, I(E) \approx 1 - \frac{|\lambda|}{\omega} \ln\left(1 - \frac{\omega}{E - E^*}\right) = 0, \tag{18}$$

an attractive interaction, $\lambda' = -|\lambda'|$ having been assumed. The first zero occurs at

$$E \equiv E_c = E^* - \frac{\omega}{\epsilon^{1/\xi} - 1} \; ; \left(\xi = \frac{|\lambda'|}{\omega} = |\lambda| \frac{\langle u^2\rangle_{av}}{\omega} ; \; u^2 = u^2(E^*)\right). \tag{19}$$

This solution is mathematically acceptable for a large range of values of ξ . Physically, we can make some interesting deductions. It is not possible to express the excitation energy $E_c - E_0(A+1)$ as we did in Eq. (8), namely as $\epsilon^0_k + \epsilon^0_\mu - \epsilon_k - \epsilon_\mu$. The excited state of the A+1 system is thus not separated from the chosen configuration by the (approximate) difference of H-F energies. Relevant to the two particle spectrum of excitations, our states E_c are not then given by $E_0(A+1) + \Delta\epsilon_k + \Delta\epsilon_\mu$. In fact, we shall argue that the lowest state of excitation in the two particle H-F spectrum lies at an energy much higher than $E_c - E_0(A+1)$.

To arrange this we need only require that $0 \leq E_c - E_o \leq 7.5$ Mev (the figure quoted for the spin-orbit splitting). On the other hand it can already be seen from Fig. 5, in which $1 + |\lambda| I(E) = 0$ is plotted versus E, that this can be guaranteed. The three parameters of Eq. (19), these being replaced by ξ , have certain numerical values. Thus the splitting off of E from the H-F spectrum is controlled by ξ and ω. Instead of estimating ξ we make interpretations of Eq. (19) based upon results obtained from more exact treatments.

The nomenclature <u>two particle bound state</u> is introduced to describe any state having the character of that $E_c - E_o$. This terminology has to do with the relative position of such states in the two particle spectrum of excitations. However, the underlying physics is not yet clarified. To see what this is we look again at the reaction channel of (5b). Both the neutron and proton are originally in shell model particle states with respect to the Be^8 ground state, as vacuum. The interaction of these nucleons with those of the core, $(1s_{1/2})^4 (1p_{3/2})^4$, can lead to core excitations. These are described in terms of shell model hole and shell model particle creation operators acting upon the physical vacuum. An alternative but equivalent representation is obtained by utilizing a new set of creation operators, those for <u>quasi-particles</u>. Such operators are formed by taking linear combinations, of appropriate nature, of those for

shell model hole and shell model particle states. A partial diagonalization of the residual internucleon force is implied by the transformation. It is this which determines the particular linear combination of operators. Although this kind of technique originates in pairing theories, the latter are not necessary for the quasi-particle picture employed here. Summation of the cluster corrections from Brueckner theory, according to a set of prescriptions will produce the additional diagonalization (over and above that giving H-F energies). To get the Be^8 parity right, we consider states with two-quasi particles present. The lowest of these is that corresponding to the presence of a single 2^+ phonon, relative to Be^8 as physical vacuum.

All of the previous remarks are summarized in Fig. 6. Notice that we imply, by Fig. (6a), an excitation of the collective state occurring as a self-energy insertion in a single particle line. In addition a distinction has been made between the shell model states of added particle (the proton) and odd target nucleon (the neutron). The former are characterized as having complex H-F energies or finite lifetimes. As such the independent excitations corresponding to these states are often, and we shall follow this usage, also called quasi-particles. This is a characteristic terminology of the Green's function treatments. (23) The finite lifetime for the neutron is neglected here. It thus has a real H-F energy. Charge exchange involving

other than valence neutrons could not be realistically (quantitatively) treated ignoring finite lifetime corrections. We require that two states, $E(k_o') \equiv \epsilon_k'$ and $E(k_o'') \equiv \epsilon_k''$, exist in the single "quasi-particle" spectrum with energy difference approximately equal to the excitation energy for a collective motion. This criterion was first given by J. Schrieffer. (5) It tells when we can expect to see a large shift in energy or lifetime, principally the latter, in the single "quasi-particle" spectrum. Physically, then, the two-particle bound state represents the renormalization of the optical potential (principally its imaginary part) due to couplings of the single quasi-particle spectrum to collective motions. (24) A hypothetical example, e.g., low energy neutron scattering on O^{15}, furnishes more graphic illustration than does our charge exchange problem. The incident neutron is already in a quasi-particle state with respect to O^{16} as physical vacuum. In another view, the neutron would be described as in a particle state with respect to the O^{15} ground state. The first, A+1 occupied, H-F states of the (n,O^{15}) system form the chosen configuration as before. Intermediate states of the A+1 system are those of two quasi-particles in one picture and those of two particles and one hole in the other, for example. At this point, the superficial differences in nomenclature vanish. We are led to look for particle summation procedure (i.e., a partial diagonalization) reproducing the O^{16} excitation spectrum of

low-lying states. This implies the quasi-particle representation. The partial summation leads to the renormalization already described. On the other nand, our inclination in a straight forward application of Brueckner theory is to convince ourselves that the third-order and higher cluster corrections are small. Precisely this point of view is responsible for our failure to treat nucleon interactions with finite nuclei in a convincing way.

Referring again to Fig. (6a), we require that $E(k_0'') \approx E(k_0) - E_x^{c*}$ where E_x^{c*} is the 2^+ excitation energy relative to the assumed vacuum. Also, $E(k_0')$ ought to be given approximately by $E(k_0) \pm \omega^*$, where ω^* is the interval in energy with which coupling to the collective motion occurs. Comparing with Eq. (19), and using $E(k_0'') - E(k_0') = E_x^{c*} \pm \omega^* = E_c$, we see that these qualitative arguments are consistent with the formal results. Asterisks on quantities are meant to imply renormalizations which cannot be dealt with in the present context. With Fig. (6c) we show how two quasi-particles of the Be^8 core can interact over large distances by exchanging a vibration. This long range interaction is not generally available to our n-p system. The latter is not imagined, here, to form a part of a collectively excited group of nucleons. Only the core nucleons satisfy this requirement. A long-range interaction involving the n-p system is shown in Fig. (6d).

We are now in a position to add the t-matrix, $\tilde{t}$, of (19) to that whose matrix elements are implied by (11). The only difficulty arises in obtaining the relevant bound state operator. Referring now to Fig. (6b) we write

$$\langle \tilde{t}(E) \rangle_0 = -i\pi\rho(E_{k_0} - E_x^c)|(\Lambda)_{av}|^2 \frac{1}{E_x^c} \hat{t}(E_{01};\omega). \qquad (20)$$

The symbols have the following meaning: $\rho(E_{k_0}-E_x^c) \approx \rho(E_{k_0})$ is the density-in-energy of two-particle bound states at the energy of the incident proton; $(\Lambda)_{av}$ is the strength of the two nucleon, phonon vertex; $\hat{t}$ is the t-matrix for a system of two nucleons with lab. energy E_{01} evaluated at the momentum transfer associated with the energy ω. Every quantity in (20) can be estimated. In particular, apart from statistical factors and others for dimensions, $|\hat{t}(E_{01};\omega=0)|^2$ is just the two-nucleon laboratory, total elastic cross section, $E_{01} = E(k_0)+E(k_1)$, at energy E_{01}. The strength of the vertex function is determined when we construct the excitation spectrum of our core, Be^8. Fig. (6b) is relevant to this computation. Estimates of $\rho(E_{k_0})$ can be made using simple thermodynamic arguments. (25) The importance of the density of states is that it determines the energy variation of the bound state t-matrix. The resulting sum of t-matrices which we have constructed can give rise to an interference structure in the (He^3,T) cross section. The matrix t is non-hermitian. In appending t to the matrix element $\langle\tilde{t}\rangle_0$ in the chosen configuration, the rapidly varying part of

the residual force has been accounted for. Presumably, b_{oo} is of slower variation. It, b_{oo}, is not however a residual force. No discussion has been given as yet of the proton H-F field. There are questions attendant with the simple-minded addition of t-matrices. These have to do with whether the sum of residues corresponding to poles in the resulting two-particle operator comes out correctly. Generally, the answer is a categorical, no! The difficulty is ignored here so as to admit simple, relevant physical details. Actually, we should prefer to recast the problem in terms of an alternative many-body formulation. Here one would use the C^{12} ground state as the physical vacuum. It is then possible to project out the initial, $Be^{9}+He^{3}$, and final, $T+B^{9*}$, configurations, from C^{12*} states. The overlap between these would then be determined by the composition of C^{12*} states. Again we should encounter after tedious algebraic manipulations the physical features described here. On the other hand, the presentation would have <u>appeared</u> less heuristic.

III. Details for 2-body t-matrix

In the preceding section it was suggested that: (i) the matrix element of the two-body transition operator in the chosen configuration could be expressed as $b_{oo}+\langle\tilde{t}\rangle_{o}$; (ii) the t-matrix could be written as a sum of diagonal and

and non-diagonal terms, $t=\bar{t}+I$. We wish to examine the relationships between these statements. In regard to (i), it is necessary to put Eq. (11) on a quantitative basis. Also, the internucleon potential, v, must be specified for the physical (charge-exchange) problem at hand. This problem through the two-body matrix element of Eq. (4) places restrictions upon any formal results. The formal statements, relating mostly to (ii), will appear as a set of rules. These rules will pertain to how we interpret and compute $t(\equiv t_{01})$. The physical restrictions tell us what procedures are likely to yield reliable numbers. They also present certain concrete aspects to be dealt with. Among these is the nearly axiomatic statement: The two-body t-matrix appearing as the result of interactions between complex systems is always off the energy shell. (This is, for example, one of the difficulties encountered in impulse approximation descriptions of elastic n-d scattering). There is further the related problem that one is thereby instructed to keep the H-F potential out of the transition-inducing part of the interaction matrix element. This relates to the use of nuclear distortions and their proper incorporation. We have alluded to this aspect in a qualitative way thus far. Finally, some attention ought to be given those formal aspects having to do with the addition of t-matrices. It is actually unambiguous, following the inclusion of the formal details. The foregoing list then comprises the topics of this section.

To get at the decomposition of t-matrices implied by (i) and (ii), let us restate the rules governing such. This is done within the framework of many-particle theory and the H-F method. The analyses previously carried out by Bethe (26) and Shaw (27) form the basis of our discussion. In adding a particle to a ground state nuclear system, it is necessary that the H-F energy operator for the particle be diagonal. In Brueckner theory, the interaction v_{oj} $(j=1,\ldots,A)$ between particle and target is eliminated in favor of t_{oj}. We have used a version of the integral equation, (7'), which relates the operators t and v to each other. In computing the energy of the chosen configuration in A+1 particles and then subtracting the ground state energy $E_o(A)$, that for the target, one encounters the matrix elements $\sum_j (n_o^o n_j^o | t_{oj} | n_o^o n_j)$. If we define t to be a sum of diagonal and non-diagonal operators, then clearly

$$(n_o^o n_j^o | I_{oj} | n_o^o n_j^o) = 0 \text{ , or } (n_o^o n_j^o | t_{oj} - \bar{t}_{oj} | n_o^o n_j^o) = 0.$$

We require, following Watson, that $\bar{t}_{oj}$ be diagonal in nuclear states. Thus, in forming the energy operator $T_o + \sum \bar{t}_{oj}$ for the added particle, neither the k.e. operator nor the average interaction operator is separately diagonal in nucleon states. It is however possible to diagonalize the operator sum $h(o)$. Because T_o is a one-body operator, $\bar{t}$ can only change the state of the added particle. However, again these non-diagonal matrix elements are restricted according to

$$(n_o' \mid T_o \mid n_o) + \sum_j (n_o' n_j \mid \bar{t}_{oj} \mid n_o n_j) = 0.$$

Bethe points out that we are to impose the equality

$$(n_o' n_j \mid \bar{t}_{oj} \mid n_o n_j) = (n_o' n_j^o \mid t_{oj} \mid n_o n_j^o)$$

even if $n_j \neq n_j^o$. We readily find from this statement, $n_j \to n_j^o$, that

$$(n_o' n_j^o \mid I_{oj} \mid n_o n_j^o) = 0.$$

Also note that because of the diagonality of $\bar{t}_{oj}$, we have

$$(n_o^o n_j \mid I_{oj} \mid n_o^o n_j^o) = (n_o^o n_j \mid t_{oj} \mid n_o^o n_j^o).$$

It is altogether clear that

$$(n_o' n_j' \mid I_{oj} \mid n_o^o n_j^o) = (n_o' n_j' \mid t_{oj} \mid n_o^o n_j^o).$$

The rules which follow are: I_{oj} excites j alone or simultaneously "0" and j; $\bar{t}_{oj}$ excites "0" alone; t_{oj} can excite either or both nucleons. We have tacitly assumed throughout that our matrix elements are antisymmetrized.

These simple results enable us to put Eqs. (7) - (11) on a quantitative basis. The two-state approximation of the earlier discussion is given in a valid manner here. To this end the relevant single particle (j-j) states and the occupied state for added nucleon are shown in Fig. 7. The bound states are generated by employing an oscillator approximation to the self-consistent well. Yet we are restricted by the c.f.p. representation which led to Eq. (4). The $\ell^n, (1p)^5$, configuration used for the bound states, μ,

implies that the individual values of total angular momentum and its 3rd component, (j,π_j), are not good quantum numbers. This is expressed as

$$|\mu(1/2s;n\ell m)\rangle = \sum_{j\pi_j} \left[\begin{matrix} 1/2 & \ell \\ s & m \end{matrix} \middle| \begin{matrix} j \\ \pi_j \end{matrix} \right] |\mu(nj\pi_j)\rangle .$$

It is possible to talk about transitions between the various (j-j) states. Generally speaking this feature makes available smaller energy denominators. We shall use as our two states $\mu_0 = 1p_{3/2}$ and $\mu_1 = 1p_{1/2}$. The projection operators on to these states have numerical values given by the v.c. coefficients. Thus in any two-body matrix element, a pair of such coefficients is implied.(6) In computations this means that for each value of ℓ, a pair of operators, $\Lambda_\ell^{(+)}$ and $\Lambda_\ell^{(-)}$ are introduced. The operators are connected by the relation $1 = \Lambda_\ell^{(+)} + \Lambda_\ell^{(-)}$. Our two-body matrix elements are always in the same $(n\ell)$-shell and of the general form.(7)

$$\langle \mu_\ell k' | v | \mu_\ell k \rangle$$

or

$$\langle \mu_\ell k' | v | \mu_\ell k'' \rangle \, 1 \, \langle \mu_\ell k'' | v | \mu_\ell k \rangle .$$

By operating to the right and left with the operators $\Lambda_\ell^{(\pm)}$, observing that the identity

$$\langle k' \mu_\ell | v | \mu_\ell k \rangle = \langle k' (\lambda_\ell^+ \mu_{\ell_+} + \lambda_\ell^- \mu_{\ell_-}) | v | (\lambda_\ell^+ \mu_{\ell_+} + \lambda_\ell^- \mu_{\ell_-}) k \rangle$$

holds, and that the operators $\Lambda_\ell^{(\pm)}$ are idempotent, $\Lambda_\ell^2 = \Lambda_\ell$, any combination of states can be obtained. Thus, for example,

$$M = (\Lambda_\ell^{(+)})^* \langle \mu_\ell k' | v | \mu_\ell k \rangle (\Lambda_\ell^{(-)})^+ = \langle k' \lambda_\ell^+ \mu_{\ell_+} | v | k \lambda_\ell^- \mu_{\ell_-} \rangle ;$$

$$M = \langle \Lambda_\ell^{(+)} \mu_\ell \Lambda_\ell^{(+)+} k' | \Lambda_\ell^{(+)} v \Lambda_\ell^{(-)+} | \Lambda_\ell^{(-)} \mu_\ell \Lambda_\ell^{(-)+} k \rangle ; \Lambda_\ell^{(+)} v \Lambda_\ell^{(-)+} = \tilde{v} \quad ;$$

$$\text{Tr } M = \lambda_\ell^{(+)2} \lambda_\ell^{(+)2} \langle \tilde{\mu}_\ell^{(+)} k' | \tilde{v} | \tilde{\mu}_\ell^{(-)} k \rangle = \lambda_\ell^+ \lambda_\ell^- \langle k' \mu_{\ell_+} | v | k \mu_{\ell_-} \rangle \text{ Tr } 1 ;$$

$$\langle \tilde{\mu}_\ell^{(+)} k' | \tilde{v} | \tilde{\mu}_\ell^{(-)} k \rangle = \langle \mu_\ell k' | v | \mu_\ell k \rangle ; \text{ Tr } 1 = 2$$

$$\langle k' \mu_{\ell_+} | v | \mu_{\ell_-} k \rangle = \frac{1}{2} \lambda_\ell^+ \lambda_\ell^- \langle \mu_\ell k' | v | \mu_\ell k \rangle .$$

Identifying μ_+ as μ_1 , μ_- as μ_0 and the $\lambda_\ell^\pm$ as v.c. coefficients, we see that the statement is true. The matrix elements for a change of state in j appear in the formal analysis. These are explicitly defined in terms of the matrix elements for no change of ℓ .

In so far as possible it is desirable to follow the formulation of the Brueckner theory. To emphasize that the single particle energies are computed self-consistently, we write E_k as the energy corresponding to a state $|k\rangle$. No change of notation is required for the ϵ_μ's as they already carry this implication. The integral equation for t, operating on the chosen configuration, is

$$t|0\rangle = v|0\rangle - i\pi \sum_\mu^Q \rho_{k'} (E_k^0 - E_{x\mu}) v|\mu k'\rangle \langle \mu k'| I |0\rangle + P \sum_{\mu k'}^Q \frac{v|\mu k'\rangle \langle \mu k'| I |0\rangle}{E_k^0 + \epsilon_{\mu_0} - \epsilon_\mu - E_{k'}} . \qquad (21)$$

This equation replaces that of (7') and it is to be understood that the matrix elements are antisymmetrized. Energy conserving transitions involving a change of state for both nucleons are explicitly represented by the first term. Here the density of states, in energy, is $\rho_{k'}(E_{k'}) = k'^2[\Omega/(2\pi)^3] \cdot (dk'/dE_{k'}) \int d\Omega_{\underset{\sim}{k}'}$. The quantization volume is Ω and $E_{x\mu} = \epsilon_\mu - \epsilon_0$ is the nuclear excitation energy. Conservation of energy requires that $E_{k'} = E_k^0 - E_{x\mu}$; $(\hbar k'^2/2M^*) = E_{k'}$, M^* being the effective mass. The index Q on a summation means that the chosen configuration is not to appear as an intermediate state. It is thus clear why the transition operator I appears. The form chosen for Eq. (21) has its justification in a choice to employ regular potentials, v. Therefore, t will be given in terms of an iteration on v.

For the moment, the antisymmetrization between added nucleon and target constituents is given up. This can be taken to mean that the principal value (P.V. or P) term of (21) can be rewritten.

$$P\sum = P\sum_{\mu k'} |k'\rangle \frac{v|\mu\rangle\langle\mu|I|0\rangle}{E_k^0 + \epsilon_{\mu_0} - \epsilon_\mu - E_{k'}} \langle k'|$$

$$= P\sum_{\mu} \frac{v|\mu\rangle\langle\mu|I|0\rangle}{E_k^0 + \epsilon_{\mu_0} - \epsilon_\mu - h(0)} \; ; h(0) = T_0 + U_0 \; ; |k'\rangle = |\eta(\underset{\sim}{k}'; \underset{\sim}{r}_0)\rangle. \tag{22}$$

The one-body distorting potential which acts upon the added or external nucleon is $\mathcal{U}_0$. Although particle states have finite lifetimes in Brueckner theory, it is sometimes convenient to ignore this. The computation of excitation

energies is very often characteristically done in such a manner. The scattering boundary conditions and related lifetime aspects associated with t, distinguish it from the hermitian K-matrix of Brueckner. We assume that $E_{k'}$ is real, hence $\mathcal{U}_0$ is hermitian. The added particle is distorted by its motion in the self-consistent potential $\mathcal{U}_0$. The functions η carry the real phase shifts associated with this potential. In practice we often have the complex phase shifts, δ_c, available. Use of $\delta_c(k')$ together with real $E_{k'}$ constitutes a small-width-of-the-line approximation (Im $t \ll$ Re t; Re $t \approx K$).

Next, the one-particle functions are constructed as

$$\langle \mu' | t | 0 \rangle = \langle \mu' | v | 0 \rangle - i\pi \sum_{\mu}^{Q} \rho_{k'} (E_k^0 - E_{x\mu}) \langle \mu' | v | \mu k' \rangle \langle \mu k' | I | 0 \rangle + P \sum_{\mu}^{Q} \frac{\langle \mu' | v | \mu \rangle \langle \mu | I | 0 \rangle}{E_k^0 + \epsilon_{\mu_0} - \epsilon_{\mu} - \hbar(0)} . \tag{23}$$

It is to be noted that the non-conserving transitions are ostensibly brought about by single changes of state for the bound nucleon alone. Of course this is not true as we shall see shortly in our evaluations. Such processes comprise a very small part of the number of non-conserving transitions. According to the two-state model both (μ, μ') take on the values μ_0 and μ_1. However, because of the appearance of I, μ is throughout restricted to μ_1. The two equations arising from (23) are

$$\langle \mu' | t | 0 \rangle = \langle \mu' | \hat{v} | 0 \rangle + P \frac{\langle \mu' | v | \mu_1 \rangle \langle \mu_1 | I | 0 \rangle}{E_k^0 - E_{x1} - \hbar(0)}$$

or

$$\left.\begin{aligned} f_0 &= g_0 + U_{01}\,\frac{1}{E_k^0 - E_{x_1} - \hbar(0)}\,f_1 \\ f_1 &= g_1 + U_{11}\,\frac{1}{E_k^0 - E_{x_1} - \hbar(0)}\,f_1 \end{aligned}\right\}. \qquad (24)$$

The potential $\hat{v}$ has been formed by taking the energy-conserving terms together with v. One-body amplitudes now appear for the external nucleon in interaction with the target, leaving the latter in definite state. These are the functions f_0 and f_1. A Tamm-Dancoff description of the system would be given by the expansion $\sum f_\mu |\mu\rangle$. The amplitudes g_0 and g_1 arise from $\hat{v}$ and are quasi-Born amplitudes. We identify U_{01} and U_{11}, involving v, as the pseudo-potentials acting upon the external particle. This nomenclature just means that two-particle potential is weighted by the density in one of the particles. The integral of the potential over the density produces a potential (pseudo-potential) for the remaining particle. The set of equations (24) should be compared with those (10) which arose in our qualitative discussion. We find as our solution to (24)

$$\langle \mu_0 | t | 0 \rangle = \langle \mu_0 | \hat{v} | 0 \rangle + \langle \mu_0 | v | \mu_1 \rangle \frac{1}{E_k^0 - E_{x_1} - \hbar(0) - U_{11}} \langle \mu_1 | \hat{v} | 0 \rangle .$$

Evidently we shall operate upon this expression from the left with some state $\langle k|$. Moreover, as an approximation, the non-diagonal elements of U_{11}, in the k-basis, are taken equal to zero. The result of the foregoing is to give

$$\langle k\mu_0|t|0\rangle = \langle k\mu_0|\hat{v}|0\rangle + \sum_{k'}^{Q}\langle k\mu_0|v|k'\mu_1\rangle \frac{1}{E_k^0 - E_{x_1} - E_{k'} - \langle k'\mu_1|v|k'\mu_1\rangle + i\epsilon}\langle k'\mu_1|\hat{v}|0\rangle. \quad (25)$$

We now require that all the matrix elements be antisymmetrized again. This is a way of saying that we have constructed an approximation to the solution appearing in an antisymmetrized theory. We shall not say how good the approximation is, but simply note that the structure of the t-matrix is physically satisfactory.

In so far as it is possible to approximate the matrix elements of $\hat{v}$ by those of v, (25) gives an explicit solution for t. That operator has diagonal and non-diagonal elements. Both of these are found. It is our hope that I is small. We can be more explicit. Now at least for $k=k_0$ where we go back to the chosen configuration, there obtains

$$\langle 0|\hat{v}|0\rangle = \langle 0|v|0\rangle - i\pi\rho_{k'}(E_k^0 - E_{x_1})\langle 0|v|k'\mu_1\rangle\langle k'\mu_1|t|0\rangle.$$

The Brueckner (28) version of our t-equation,

$$t = v + v\frac{Q}{e_x + i\eta}t \;;\; (v = v_{01}, t = t_{01}), \quad (26)$$

would define the excitation energy as

$$e_x = t_{11} + t_{00} + T_0 + T_1 - \mathcal{E}_0 - t_{00}^{-av} - t_{11}^{-av}. \quad (27)$$

Additional notation has been introduced here. The labels 0 and 1 refer to the interacting pair; $\mathcal{E}_0$ is the energy of the chosen configuration defined such that $\mathcal{E}_0 + t_{00}^{-av} + t_{11}^{-av} = E_0(A) = E_k^0 + \epsilon_{\mu_0}$, $E_k^0 = (\hbar^2 k_0^2/2M) + t_{00}^{-av}$, $\mathcal{E}_0 = (\epsilon_{\mu_0} - t_{11}^{-av}) + \hbar^2 k_0^2/2M$, the sum of k.e. eigenvalues for the two nucleons;

$\bar{t}$ still implies no change of state, thus $t_{00}^{-av} = \sum_{j=1}^{A} (n_0^0 n_j^0 | t | n_0^0 n_j^0)$ and $t_{ii}^{-av} = \sum_i (n_1^0 n_i^0 | V | n_1^0 n_i^0)$, (i = 1,..., (A-1)), the m.e. being antisymmetrized; t_{ii} is the full interaction operator for particle i with no charge of state for this constituent. Two points are relevant here. First, the energy of the added particle, in the chosen configuration, can be expressed as $(\hbar^2 k_0^2/2M) + t_{00}^{-av}$. Second, the perturbation development of (26) discloses that no second-order terms occur in t_{01}, leading back to the chosen configurations. Then, using (26), and writing

$$\langle 0|\hat{v}|0\rangle = \langle 0|v|0\rangle - i\pi\rho_{k'}(E_k^0 - E_{x\mu_1})\Big[\langle 0|t|k'\mu_1\rangle\langle k'\mu_1|t|0\rangle - \langle 0|v\frac{Q}{\omega_x^{(+)}}t|k'\mu_1\rangle\langle k'\mu_1|t|0\rangle\Big],$$

it immediately follows that

$$\langle 0|\hat{v}|0\rangle = \langle 0|v|0\rangle + i\pi\rho_{k'}\, E_k^0 - E_{x\mu}\, \langle 0|v\frac{Q}{\omega_x^{(+)}}t|k'\mu_1\rangle\langle k'\mu_1|t|0\rangle.$$

For the present theory, the terms now correcting v_{00} are analogous to the cluster corrections. Their specific form is different from that ordinarily encountered. This is explained as arising from the c.f.p. representation. To a very good approximation $\hat{v}_{00} \approx v_{00}$ when the diagonal elements of $\bar{t}$ are computed from (25). It is still necessary to compute $\langle k'\mu'|\hat{v}|0\rangle$ which contains still another matrix element of t, namely that of I. We know something about the non-diagonal elements of t from the restriction

$$(n_0'|T_0|n_0^0) + \sum_j^A (n_0' n_j^0|\bar{t}|n_0^0 n_j^0) = 0,$$

Again thinking in terms of a c.f.p. representation, where ℓ^n determines the effective A to be A = n, it is evident that

$$\frac{1}{5}\sum_{1}^{5}\bar{t} \equiv (n_0' n_1^0 |\bar{t}| 0)_{av} \approx -\frac{1}{5}(n_0' |T_0| n_0^0); (\ell^n = (1p)^5). \tag{28}$$

This means that in most practical applications, the non-diagonal elements of $\bar{t}$ are going to be quite small. We still however need an estimate of I in order to solve Eq. (25) explicitly. To get this, we make the usual small-wiath-of-the-line approximation. In an algebraic operator notation, we write

$$\hat{v} = v - i\pi\rho v t \; ; \; \mathrm{Re}t = \alpha, \; \mathrm{Im}t = \beta \; ; \; \text{and} \tag{23'}$$

$$t = \hat{v} + v\,\frac{P}{D_Q}\,t .$$

$$\mathrm{Re}t = v + \pi\rho v\beta + v\,\frac{P}{D_Q}\,\mathrm{Re}t$$

$$\approx v + \pi\rho v\beta + v\,\frac{P}{D_Q}\,v = \mathbf{X} + \pi\rho v\beta$$

$$\mathrm{Im}t = -\pi\rho v\alpha + v\,\frac{P}{D_Q}\,\mathrm{Im}t$$

$$\approx -\pi\rho v\alpha + v\,\frac{P}{D_Q}(-\pi\rho v\alpha) = -\pi\rho\mathbf{X}\alpha$$

$$t \approx \mathbf{X}(1 - i\pi\rho\mathbf{X}); \; \mathbf{X} = v + v\,\frac{P}{D_Q}\,v. \tag{29}$$

All of the matrix elements of I are computed from this equation. Consequently $\hat{v}$ is given in terms of v and finally the diagonal elements of $\bar{t}$ are given from (25). We may also note that the I matrix elements of (29) are

improved by writing $t = X(1+i\pi\rho X)^{-1}$ The crude procedure introduced here does lead to a full determination of the two-body t-matrix. It is suitable to regular potentials which can lead to finite values of the iterated form of X.

The evaluation of (25) can be completed now. There was in our beginning equations a P.V. such that $E_k^0 - E_{\alpha_1} - E_{k'} \neq 0$ held. This restriction is automatically satisfied now and the P.V. notation dropped. A small imaginary part, $+i\epsilon$ (used interchangably with the notation, $+i\eta$), has been added into the energy denominator. This reflects the boundary conditions on t. The matrix element $\langle k'\mu_1 | v | \mu_1 k' \rangle$ can be approximated by the real part of the optical potential for elastic scattering at the energy $\hat{E}_{k'} = \hbar^2 k'^2/2M$. A rough estimate of the energy dependence is given by $v_{111}(k') = \hat{E} + 0.5\,\hat{E}_{k'}$, $\hat{E} = -40$ MeV. (29) The effective mass approximation yields $E_{k'} = (M/M^*)\hat{E}_{k'}$ where the ratio is roughly $(0.7)^{-1}$. It is then apparent that the energy denominator vanishes for fairly large $E_{k'}$, $E_{k'} = (1.35)^{-1}\cdot(E_k^0 - E_\alpha - \hat{E})$. In the example given, (see Fig. 7), this condition obtains for $E_{k'} = 35.2$ MeV or $\hat{E}_{k'} = 24.8$ MeV. The corresponding value of E_k^0 is 23 MeV for 14.5 MeV laboratory protons (1.7 MeV of excitation energy is added to the lab. energy). Even a fairly soft potential v will have appreciable momentum components for the energies quoted. However, the point is the matrix elements of v and $\hat{v}$ are

appreciably cut down. In particular, this is true for the states required in the physical problem. For example in the m.e. $\langle k'\mu_f|\hat{v}|0\rangle$, with the numerical values cited above, we ask for the correlation of momentum components in $\hat{v}$ separated by some 19 Mev.(8) The components are taken from those lying in the intervals about 23 and 42 Mev. Our statement is only semi-quantitative as one has yet to consider the c.m. and relative energies of motion in the two-particle system. At any rate we shall be able to see for conventional potentials v that the matrix elements of t are mostly given by the first term of (25). Again, only the diagonal elements of t are computed from this equation. The non-diagonal elements are, we indicated, gotten from (29). It is also true that (28) gave a restriction which must be observed to get an H-F representation. The P.V. terms of (25) are likewise expected to be small. In a rough way, contributions from states $E_{k'} < 35$ Mev tend to cancel against those witn $E_{k'} > 35$ Mev. The potential v and also that $\hat{v}$ will not in general have appreciable momentum components beyond an energy corresponding to $E_{k'} = 90$ Mev. These remarks and others made here serve to emphasize the off-energy snell character of Eq. (25). The problem is more extensive than this. And, this remark has to do with the restrictions imposed upon t by the m.e. of (4).

It should be our point of view, regarding Eq. (4), that the two-body matrix element is computed according to prescriptions from many-body theory. These have been discussed in connection with (25). The procedure by which the spatial 2-body m.e. of (4) is reduced to one calculable from many-body theory is straightforward. It is only necessary to require the Lorentz invariance of t_{01} matrix elements. We use this invariance here. Møller (30) has given the invariance relation as

$$f(W_{12}\, W^0_{12}) \langle p_1 p_2 | t_{12} | p^0_1 p^0_2 \rangle = f(\bar{W}_{12}\, \bar{W}^0_{12}) \langle \bar{p}_1\, \bar{p}_2 | \bar{t}_{12} | \bar{p}^0_1\, \bar{p}^0_2 \rangle . \qquad (30)$$

The quantities p_i are four-vectors, $p_i = (\underset{\sim}{p}_i, E_i)$ where the energies E_i are the corresponding kinetic energies for the particles i. The factors f are explicitly $f = \sqrt{W_{12} W^0_{12}}$, the positive root being taken; $W_{12} = W_1 W_2$ is the product of total energies for the interacting particles. Each p_i is carried into $\bar{p}_i$ through the Lorentz transformation $Lp_i = \bar{p}_i$. Similarly, the t-matrix transforms according to $\bar{t} = LtL^{-1}$. We choose the translation

$$L\, \underset{\sim}{p}_i = \underset{\sim}{\bar{p}}_i = \underset{\sim}{p}_i - \underset{\sim}{u}$$

as our transformation. The vector $\hat{\underset{\sim}{u}}$ is chosen so as to give particle "1" of Eq. (4) zero momentum in the initial state; thus $\hat{\underset{\sim}{u}} = \hat{\underset{\sim}{k}}_0$. We have gone into a frame in which the bound nucleon is at rest. This is by definition its laboratory system. It is straight forward to show that the m.e. of (4), apart from factors of f's, is replaced by

$$\left\langle \psi(\pi_1) F_{\chi_0}(s_0)\, e^{i\underset{\sim}{\zeta}_F\cdot\underset{\sim}{z}_1} \mid \tau_{01} \mid \psi(\pi_0)\, F_{\chi_1}(s_1)\, e^{i\underset{\sim}{\zeta}_0\cdot\underset{\sim}{z}_0} \right\rangle = g \equiv (t_{01})_{m.e.};$$

$$\underset{\sim}{\zeta}_F = (\underset{\sim}{k}_F + 2/3\underset{\sim}{q}_\chi - \underset{\sim}{\gamma}) + \underset{\sim}{q}/27;\ \underset{\sim}{\zeta}_0 = -(\underset{\sim}{k}_0 + 2/3\underset{\sim}{q}_\chi - \underset{\sim}{\gamma}). \qquad (31)$$

One has written τ_{01} for the operator occurring in the matrix element. This is quite correct. It is understood that this is the operator giving rise to transitions within the specified laboratory system. In particular it is this operator whose matrix elements are computed in the product representation of the two-state model. We have yet to obtain the operator which our many-body analysis applies. The Lorentz transformation factor, ξ , accompanying (31) is

$$\xi(\underset{\sim}{k}_0, \underset{\sim}{k}_F; \underset{\sim}{\gamma}) = \left[\frac{y(\lambda_c \zeta_0)\, y(\lambda_c \zeta_F)}{y(k_0^* \lambda_c)\, y(\omega_0 \lambda_c)\, y(\omega_F \lambda_c)} \right]^{1/2} \qquad (32)$$

$$y(x) = 1 + 0.5x^2,\ \lambda_c = \hbar/Mc,\ k_0^* = k_0/27.$$

This factor has been given in the non-relativistic limit and will not differ appreciably from unity at a bombarding energy of $E_{lab}(k_0) = 25$ MeV. This is true for all scattering angles $\vartheta(\hat{\underset{\sim}{k}}_F \widehat{\ } \hat{\underset{\sim}{k}}_0)$. So, we expect ξ to be insensitive to the momentum transfer $\underset{\sim}{q}$. Although $\hat{\underset{\sim}{\gamma}}$ is integrated over all values $|\hat{\underset{\sim}{\gamma}}|$, the form factor entering (4) selects a range of values about some γ_0. Within this range: a) the n.r. approximation to ξ holds, and; b) ξ is essentially equal to unity. The matrix element replacing that of (4) is now

$$(F|t_{01}|0) = \int \frac{d\underset{\sim}{\gamma}}{(2\pi)^3}\, G^*_{J\varkappa_2'}(\underset{\sim}{\gamma})\, G_{J\varkappa_2}(\underset{\sim}{\gamma} - 2/3\underset{\sim}{q}_\chi)\, \xi(\underset{\sim}{k}_0, \underset{\sim}{k}_F; \underset{\sim}{\gamma})\, g(\underset{\sim}{k}_0, \underset{\sim}{k}_F; \underset{\sim}{\gamma} - 2/3\underset{\sim}{q}_\chi). \qquad (4')$$

The last factor, namely g, has been written in a manner which suggests the neglect of off-energy-shell effects.

Strictly speaking, (3), (4) and (31) show that we are always off of the energy shell. This continues to be true for ground-to-ground transitions of elastic scattering where $|\hat{\underset{\sim}{k}}_0| = |\hat{\underset{\sim}{k}}_F|$. For charge exchange, the ground-to-ground transition does not imply the equality. (We cite, for example, a Q-value of -1.09 Mev for the $Be^9(He^3,T)B^9$ ground state transition). We intend to exploit this fact. Relevant to the m.e. of (31), the salient feature is that the same bound state orbital appears in both initial and final states. Nucleons in this orbital are bound with some effective energy B'. Transitions are however induced between scattering states of energy $E(\zeta_0^2)-B'$ and $E(\zeta_F^2)-B'$. We shall return to this aspect subsequent to a discussion of (4'). It is our intention to ignore the $\hat{\underset{\sim}{\gamma}}$-dependence in g. This is done by writing $\hat{\underset{\sim}{\gamma}} = (\hat{\underset{\sim}{k}}_0/k_0)\gamma_0$ where γ_0 measures, for given $(\mathcal{L}_2', \mathcal{L}_2)$, the predominant momentum component for a nucleon in the form factor. The determination of γ_0 has been discussed previously. In effect, we reduce $E_0(k_0)$ by an amount $E_b(\gamma_0)-2\sqrt{E_b E_0}$, which for 2 Mev of binding of a single nucleon in the 3-body system amounts to 12 Mev. The 2 Mev is an effective value of E_b determined by the form factor of (4'). It can be argued for the problem at hand that $E_b \approx 2$ Mev is about correct. Our previous qualitative arguments that $E_0(k_0)-B$ should be about 15 Mev thus hold.

The approximation to (4') is then

$$(F|t_{01}|0) = \int \frac{d\underset{\sim}{\gamma}}{(2\pi)^3} G^*_{\mathcal{H}_2'}(\underset{\sim}{\gamma}) G_{\mathcal{H}_2}(\underset{\sim}{\gamma} - 2/3\underset{\sim}{q}) g(\underset{\sim}{k}_0, \underset{\sim}{k}_F; 2/3\underset{\sim}{q})$$
$$= F(\mathcal{H}_2' \mathcal{H}_2; \underset{\sim}{q}) g(\underset{\sim}{k}_0, \underset{\sim}{k}_F; 2/3\underset{\sim}{q})_{\gamma_0} \qquad (33)$$

This form is familiar from the usual impulse approximations, with local t-matrix, to elastic nucleon scattering. (31) In evaluations of the g-element of (33) we shall not use the diagonal part of t_{01}, namely $(\bar{t}_{01})_d$. This emphasizes the off-energy-shell aspect. Instead, $(\bar{t}_{01})_{n.d.}$, the operator changing scattering states, is to be employed in (33). Some discussion of $(\bar{t}_{01})_{n.d.}$ has appeared in connection with (28). But, this must be augmented for quantitative purposes. The H-F energy giving nuclear distortions, i.e. distorted-wave matrix elements, is computed from the sum $(\bar{t}_{01})+\tilde{t}$, (see Eq. (20)). We require no explicit knowledge of $\tilde{t}$, the non-diagonal operator, when the combined c.f.p. and cluster model representations are employed, together with regular potentials v.

We still must show how the various operators $\bar{t}_{01}$, $\tilde{t}$ are used in computations. And, in particular the relation between $\bar{t}_{01}$, $\tilde{t}$ and τ_{01} must be examined. There is an indistinguishability between the transition implied by (31) and that which we write, in i-spin and position coordinates, as

$$\langle \xi^+_{1/2}(1) F_{q_0}(1) \xi^-_{1/2}(0) e^{i\underset{\sim}{\zeta}_F \cdot \underset{\sim}{z}_0} | \tau_{01} | \xi^-_{1/2}(1) F_{q_1}(1) \xi^+_{1/2}(0) e^{i\underset{\sim}{\zeta}_0 \cdot \underset{\sim}{z}_0} \rangle = g'. \qquad (34)$$

Again, recall that $q_i = (n,\ell)_i$ gives the principal and orbital quantum numbers of the bound nucleon. In the c.f.p. representation, we shall have $q_0 = q_1$. Of course there is a difference in binding energy of the last nucleon as we go from one member of a mirror pair, e.g. (Be^9, B^9) to the other. However, this difference is reflected in the initial and final kinetic energies of the observed particles. Thus, we can consider the bound constituent to remain in some given energy state, B'. The scattering constituent has its energy measured from this value. Its relative kinetic energies are $E(\zeta_0^2)-B'$ and $E(\zeta_F^2)-B'$ in initial and final states. That $|\underset{\sim}{\zeta}_0| \neq |\underset{\sim}{\zeta}_F|$ is a reflection of the physical difference of nucleon binding energies in the mirror pairs. These qualifications permit us to use g^{1} of (34) in place of g of (31). Fig. 8 shows that g^{1} differs from g only in the manner in which the nucleons are labelled. We shall understand in what follows that our single particle functions are

$$g_0^{\nu}(1) = \xi_{1/2}^{\nu}(1) F_{g_0}(1) \quad \text{and} \quad \lambda_{\underset{\sim}{k}}^{\nu'}(0) = \xi_{1/2}^{\nu'}(0) \epsilon^{i\underset{\sim}{k}\cdot\underset{\sim}{z}_0} \; ; (\nu,\nu') = \pm 1. \qquad (35)$$

However, it is less cumbersome to carry the indices (ν,ν'). These will only be implied in the subsequent work. Our aim now is to replace (31) by a matrix element which we shall describe as being self-consistent. To this end we rewrite (31) as

$$g = \left\langle g_0 \lambda_{\underset{\sim}{k}'} \left| \left(\tau_{01}\right)_d + \left(\tau_{01}\right)_{n.d.} \right| g_0 \lambda_{\underset{\sim}{k}} \right\rangle \qquad (31')$$

The diagonal operator $(\tau_{01})d$ is defined as giving the H-F energy. The operator $(\tau_{01})n.d.$ changes only the state of the particle in the continuum. Eq. (31') must not be solved as $g = \langle g_0 \lambda_k' | (\tau_{01})n.d. | g_0 \lambda_k \rangle$ which solution would emphasize the model replacements of (35) and (34). Instead, a two-particle correlated function Ψ_k, the stationary state of interaction, is introduced as

$$\tau_{01}\, g_0(1)\lambda_k(0) = \left(U_{01} + \pi_{01}\right)\Psi_k(0,1). \tag{36}$$

The potentials U_{01} and π_{01} are respectively diagonal and non-diagonal according to definitions previously given. The function $\Psi_k^{(+)}$ satisfies an equation of the form

$$\Psi_k^{(+)} = g_0\lambda_k + \frac{1}{a_0^{(+)}}\left(U_{01}+\pi_{01}\right)\Psi_k^{(+)};\ a_0^{(+)} = E - T_0 - \hbar(1) + i\epsilon \tag{37}$$

It is imagined that $\hbar(1)$ is H-F energy operator for the bound particle. Introducing the approximation that we are on the energy shell, according to which a set of time-reversed distorted wave functions $\eta_{k'}^{(-)} g_0$ exist, we write

$$\eta_{k'}^{(-)} g_0 = g_0\lambda_{k'} + \frac{1}{a_0^{(-)}} U_{01}\, \eta_{k'}^{(-)} g_0. \tag{38}$$

The g-matrix element is then determined using the procedure of Gell-Mann and Goldberger. (32)

$$g = \langle g_o \lambda_{k'} | U_{01} + \pi_{01} | \Psi_k^{(+)} \rangle$$

$$= \langle \eta_{k'}^{(-)} g_o - \frac{1}{a_o^{(-)}} U_{01} \eta_{k'}^{(-)} g_o | U_{01} + \pi_{01} | \Psi_k^{(+)} \rangle$$

$$= \langle \eta_{k'}^{(-)} g_o | U_{01} + \pi_{01} | \Psi_k^{(+)} \rangle - \langle \eta_{k'}^{(-)} g_o | U_{01} | \Psi_k^{(+)} - g_o \lambda_k \rangle \qquad (39)$$

$$= \langle \eta_{k'}^{(-)} g_o | \pi_{01} | \Psi_k^{(+)} \rangle + \langle \eta_{k'}^{(-)} g_o | U_{01} | g_o \lambda_k \rangle .$$

It is now correct to take second term of (39) as equal to zero. On the other hand, we define the non-diagonal t-matrix, $(\bar{\bar{t}}_{01})$n.d. through the relation

$$(\bar{t}_{01})_{n.d.} \eta_k^{(+)} g_o = \pi_{01} \Psi_k^{(+)} ; (\bar{t}_{01})_{n.d.} = (\tau_{01})_{n.d.} . \qquad (40)$$

In view of these statements, we write our matrix element as

$$g = \langle \eta_{k'}^{(-)} g_o | (\bar{t}_{01})_{n.d.} | \eta_k^{(+)} g_o \rangle . \qquad (41)$$

The analogous statement for (31) is

$$g \equiv (t_{01})_{m.e.} = \langle \psi(\pi_i) F_{q_o}(s_o) \eta^{(-)}(\underset{\sim}{\zeta}_F ; \underset{\sim}{z}_i) | (\bar{t}_{01})_{n.d.} | \psi \pi F_{q_i}(s_i) \eta^{(+)}(\underset{\sim}{\zeta}_o ; \underset{\sim}{z}_o) \rangle . \qquad (31')$$

Here, again, we are off the energy shell. Nevertheless, our procedure has defined the manner in which one is to go off the energy shell. Distortions in the initial and final states are produced by the potential U_{01} whose matrix elements equal those of $(\tau_{01})_d = (\bar{t}_{01})_d + \tilde{t}$. We recall that $\tilde{t}$ contains the collective excitations.

Apart from the antisymmetrization, (31') represents a self-consistent statement of the exchange problem. We can now account for the scattering interactions of initial and final nuclear systems. With $\tilde{t}$ turned off, the phase shifts in $\eta^{(+)}$ are those for protons incident upon the target (A=9 for Be^9) at the energy $E(\zeta_0^2)$. A similar statement holds for the neutron channel $\eta^{(-)}$. In constructing the cluster representations for 3-body systems, a totally antisymmetric 3-body function is employed, or should be. Therefore, the initial A=3 system scatters, in principle, in the H-F potential generated by, here, the A=9 target. The role of $\tilde{t}$ is to describe, by (20), the collective excitations produced by the bombarding or scattering systems. We have discussed the specific nature of the collective states for $Be^9(He^3,T)B^{9*}$. Every other problem will require a similar analysis. It is relevant to observe that $\tilde{t}$ will influence $\eta^{(-)}$ for B^{9*}(Q =-3.42 Mev) quite importantly while $\eta^{(-)}$ for Be^9 will not be so seriously effected. In the entrance channel, $\eta^{(+)}$, we expect to be able to ignore $\tilde{t}$ altogether. The advantage provided by (31'), which offsets some of the formal complexity, is seen by referring to (28). In view of the latter we can express the interaction as

$$(\bar{t}_{01})_{n.d.} = -\frac{\hbar^2 k_0^2}{2M^*}\,\delta(\underset{\sim}{r}_0 - \underset{\sim}{r}_1)\,O_{e.x.}\,. \qquad (42)$$

The constant M* appears as an effective mass, but this is not to be taken literally, O_{ex} gives the exchange mixture of the force.

We specify now the exchange mixture of the force. It is fairly well established that the 2-body force is charge independent. And, for the energies of interest here, this is certainly a valid picture. On such a basis, we can write

$$v_{01} = \left[\frac{3+\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2}{4}\, v_{01}^{t} + \frac{1-\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2}{4}\, v_{01}^{s}\right] P^{\tau};\quad P^{\tau} = \frac{1}{2}\left(1+\underset{\sim}{\tau}_1\cdot\underset{\sim}{\tau}_2\right)$$

$$v_{01}^{m} = \left[(V_0)_m(\mu_m r_{01})^{-1}\exp-(\mu_m r_{01})\right] 1/2\left(1+P^{M}\right). \tag{43}$$

The spin-orbit and tensor forces have been omitted here. The latter is certainly important for the computation of binding energies. Also, the former enters the H-F interactions for both positive and negative energy states in a significant way. As P^M is the space exchange operator, (43) implies a Serber interaction. The potential parameters are: $\mu_s = \mu_t$ with $\mu_t = (1.19)^{-1}\ f^{-1}$ (inverse fermi units); $(V_0)_s = 0.6\ (V_0)_t$; $(V_0^-)_t = -40$ Mev. These are parameters suggested by the complex of low energy analyses of the two-body problem. (33) For charge exchange, we use only the part of P^{τ}, the isotopic-spin exchange operator, given by $(1/4)\left[\tau^{+}(0)\tau^{-}(1) + \tau^{-}(0)\tau^{+}(1)\right]$. The notation involving raising and lowering operators in isotopic spin is standard. This completes the specification of v_{01} , which enters Eq. (29). For O_{ex} of (42), we use a more indirect procedure, giving the same answer as (43), but having its value in an illustrative context. In the space of totally antisymmetric, two-particle wave functions, the following representation for the force, $\tilde{V}$, is useful.

$$\tilde{V} = \frac{1}{16} \sum_{ST} t_{ST} \Lambda_S^\sigma \Lambda_T^\tau \; ; \; S \equiv [S], \; T \equiv [T]. \qquad (44)$$

Sums are carried out over the $(2S+1) = [S]$ and $(2T+1) = [T]$ labels, singlet and triplet, of spin and isotopic spin. The t-matrix t_{ST} in each state is multiplied by the projection operators in spin and i-spin. Introducing the labels (e,o) giving the parity of the space states, we have

$$\tilde{V} = \frac{1}{16} \left[\Lambda_1^\sigma \, t_{11}^o \Lambda_1^\tau + \Lambda_1^\sigma \, t_{13}^e \Lambda_3^\tau + \Lambda_3^\sigma \, t_{31}^e \Lambda_1^\tau + \Lambda_3^\sigma \, t_{33}^o \Lambda_3^\tau \right]$$

The t_{ij}^Π , $\Pi = \pm 1$, are numerical functions of position, parametrized according to (i,j), e.g., range $\mu(ij)$ and depth $c_0(ij)$. Again, for a Serber interaction, using $\tilde{V} = \frac{1}{2}(\tilde{V}^e + \tilde{V}^o)$, we find

$$\tilde{V}^e = \frac{1}{8} \left[\Lambda_1^\sigma \, t_{13}^e \Lambda_3^\tau + \Lambda_3^\sigma \, t_{31}^e \Lambda_1^\tau \right]$$

as the even state force. The identities $\Lambda_3^\tau + \Lambda_1^\tau = 1$ and $\Lambda_3^\tau - \Lambda_1^\tau = P^\tau$ can be used to rewrite this expression as

$$\tilde{V}^e = \frac{1}{16} \left[(\Lambda_1^\sigma t_{13}^e + \Lambda_3^\sigma t_{31}^e) + (\Lambda_1^\sigma t_{13}^e - \Lambda_3^\sigma t_{31}^e) P^\tau \right]. \qquad (45)$$

In the spirit of charge independence, the isotopic spin states do not affect the force parameters, thus $t_{13} = t_1$, $t_{31} = t_3$. Clearly, for the physical problem only the P^τ-term contributes. Charge exchange then measures the difference between singlet and triplet interactions. Again, this conclusion holds in the space of totally antisymmetric functions. Operating upon a product function of indefinite symmetry, $\tilde{V}^e$ has the following properties for a charge exchange process

$$\tilde{V}^e \; \psi_\ell \,(\text{space})\, \psi_s \,(\text{spin})\, \psi_i \,(i\text{-spin})$$

$$= \tilde{V}^e \frac{1}{\sqrt{2}} (\psi^e + \psi^o)(\Lambda_1^\sigma \psi_1 + \Lambda_3^\sigma \psi_3) \frac{1}{\sqrt{2}} (\psi_o' + \psi_o^o)$$

$$= \frac{1}{8} (\Lambda^\sigma t_1 - \Lambda_3^\sigma t_3) P^\tau \left| \frac{1}{2} \psi^e \, (\Lambda_1^\sigma \psi_1 + \Lambda_3^\sigma \psi_3)(\psi_o' + \psi_o^o) \right\rangle .$$

The normalization has been changed in $\tilde{V}^e$ to take into account the fact that only 8 spin and i-spin states exist for charge exchange processes. The complex conjugate function which must be used here is

$$\left\langle \frac{1}{2} \psi^e \, (\Lambda_1^\sigma \psi_1 + \Lambda_3^\sigma \psi_3)(\psi_o' - \psi_o^o) \right| \; ; \; P^\tau (\psi_o' + \psi_o^o) = \psi_o' - \psi_o^o .$$

Then, the expection which obtains is

$$\frac{1}{16} \left\langle \psi^e \mid \mathrm{Tr} \, (\Lambda_1^\sigma)^2 t_1 - \mathrm{Tr} \, (\Lambda_3^\sigma)^2 t_3 \mid \psi^e \right\rangle$$

$$= \frac{1}{16} \left\langle \psi^e \mid t_1 - 3 t_3 \mid \psi^e \right\rangle .$$

Now, we compare this with (31') and (43) where, in a symbolic language, the Dirac representatives, bra and ket, are

$$\left\langle \Psi_F^e \frac{1}{\sqrt{2}} (\psi_o' - \psi_o^o)(\Lambda_3^\sigma \psi_3 - \Lambda_1^\sigma \psi_1) \right|, \text{ and } \left| \Psi_o^e \frac{1}{\sqrt{2}} (\psi_o' + \psi_o^o)(\Lambda_3^\sigma \psi_3 + \Lambda_1^\sigma \psi_1) \right\rangle$$

This is to say that <u>all</u> of the coordinates have been exchanged within the n-p system. The expectation of v_{01}, which generates to good approximation a local t-matrix t_{01}, i.e. $t_{01} \approx v_{01}$, is

$$\frac{1}{2} \left\langle \Psi_F^e \mid 3 v_{01}^3 - v_{01}^1 \mid \Psi_o^e \right\rangle .$$

This is the same sort of result as that first obtained. We measure the difference between singlet and triplet interactions in the charge exchange reaction. The apparent sign and normalization differences relative to the previous result are trivially recovered and are unimportant here. Having made the point that (45) contains the same information as (43), we express O_{ex} in the form

$$O_{ex} = (\Lambda_3^\sigma + 0.6\, \Lambda_1^\sigma)\, P^\tau \qquad (46)$$

The restriction to a Serber force is not necessary here as O_{ex} multiplies a force of zero range.

Up to this point, we have discussed the manner in which a charge exchange reaction is to be regarded as a shell-model computation. The point of departure has been such that one seeks to keep as close to the Brueckner, Bethe and Shaw formalisms as possible. This leads to a H-F self-consistent picture within which it is clearly possible to define residual interactions in an unambiguous way. For the reaction treated, the residual force induced transitions and gave the coupling of the single particle spectrum to collective nuclear motions. Actually such a result is quite general and independent of the specific model introduced here. It emerged that the H-F interaction gave rise to nuclear distortions but no transitions. This too is a general result, in view of which one is cautioned against misunderstanding the basis of procedures which may be obscured by terminology such as "the distorted-wave method".

Distortions arise as a systematic consequence of the application of the H-F method. This cannot be emphasized too strongly. We have not gotten to the point yet where we attempt to analyze the reactions between complex nuclear systems in terms of "real" (with hard cores) t-matrices. Indeed, the work of Moszkowski and Scott (34) gives an indication that such may never be necessary. At the same time that we introduced the t-matrices, in what might seem an arbitrary way, namely, one unrelated to Brueckner theory, integral equations were given for each such operator. These equations can always be examined in a quantitative way to explore questions of ambiguity.

There is one last aspect which we would like to illustrate in this section. This has to do with the change in the optical potential, or $(\bar{t}_{01})_d$, experienced by the added nucleon when the n-p system couples to a collective state through $\tilde{t}$. It is our aim to show that the 2-particle H-F energy is changed in a way which reflects itself in the transition operator. This is to say that the transition operator of (42) implies that some of the two-nucleon force has been exhausted in achieving the collective coupling. When, then, $\tilde{t}$ is added to $(\bar{t}_{01})_d$, and it is stated that the residues of the two-particle operators are conserved, we simultaneously imply a renormalization of the transition operator. We have of course accounted for this possibility by means of the M* factor of (42), $M^* \neq 0.6M$ of Brueckner theory. It is worthwhile to see why the inequality must

be imposed. We use techniques originating in the reference of Gell-Mann and Goldberger previously cited. Symbolism of algebraic nature will be taken from the many-particle Green's function theory of Martin and Schwinger. (35) An exact analogy with that theory does not hold here, as we shall point out. It is nevertheless possible to construct the required analogy. The equation for the 2-particle stationary state of (37) implied that there existed states ψ_β diagonalizing the energy operator $H_o = h_b + T_o + U_o$; ($h_b = h(1)$, $U_o = U_{o1}$). We rewrite the total energy operator as[9]

$$H = \tilde{H}_o + \tilde{\pi}_{o1} \equiv H_o + \pi_{o1} = (H_o + \Delta_c) + (\pi_{o1} - \Delta_c). \qquad (47)$$

Transitions now occur through an operator $\tilde{\pi}_{o1}$. If we call the stationary state of the 2-nucleon system Ψ_o, then the average value of $\tilde{\pi}$ in Ψ_o is

$$\langle \tilde{\pi} \rangle_o = \langle \Psi_o | \tilde{\pi} | \Psi_o \rangle = \langle \Psi_o | t' | \Psi_o' \rangle. \qquad (48)$$

We imply that ψ_o' is the lowest physical state of the ψ_β' diagonalizing G_o', defined below. Introducing the propagators, or Green's functions G_o and G, at some energy E,

$$G_o^{-1} = E - H_o = G_o'^{-1} + \Delta_c; \; G^{-1} = E - H_o - \sum = G_o'^{-1} - \sum', \qquad (49)$$

where $G^{-1} \Psi_o = 0$, thus E is the ground state energy E_o, we imply that

$$G_o'^{-1} G = \tilde{\pi} G + 1. \qquad (50)$$

In the above, $\sum$ is the proper self-energy for the two-nucleon system. Eq. (50) is to be compared with the G equation of Kadanoff and Martin, (36)

$$G_0^{-1}(1)\, G_2(12,1'2') = \delta(1-1')\, G_1(2-2') - \delta(1-2')\, G_1(2-1') + iV\, G_3(123,1'2'3^+)\Big|_{3=1}, \tag{51}$$

where G_2 is a 2-particle function, analogous to our G, G_1 is a one-particle propagator and G_3 that for 3-particles. We do not in any sense work within the hierarchy of many-particle functions here. Eq. (50) can be rewritten as

$$\sum' G = \tilde{\pi}\, G \tag{52}$$

which is an obvious result, having the expected physical significance. It is of interest to compute the change of normalization as we go from the function Ψ_0 to that ψ_0'. However, we first note that (52) and (48) together imply that

$$\tilde{\pi}\, \Psi_0 = t'\, \psi_0' \quad \text{or} \quad \tilde{\pi}\left(\psi_0' + G\, \tilde{\pi}\, \psi_0'\right) = t'\, \psi_0'.$$

Then from

$$\tilde{\pi}(1 + G\, \tilde{\pi}) = t', \quad \tilde{\pi}\, G\left(G^{-1} + \tilde{\pi}\right) G_0' = t'\, G_0',$$

we infer by approximation to (52) that

$$\sum' G = t'\, G_0'. \tag{53}$$

This provides a prescription from which $\sum'$ may be computed. A similar result is obtained by Baym and Kadanoff (37) and

the necessary details are to be found there. The t-approximation giving $\sum'$ is often quite useful in practice. And we have used this approximation throughout our analysis. The reason for computing $\langle \Psi_0 | \psi_0' \rangle$ is that our approximation to (48), namely (31'), involves writing

$$\langle \tilde{\pi} \rangle_0 \approx \langle \psi_0' | t' | \psi_0' \rangle. \tag{54}$$

We have then

$$\langle \Psi_0 | \psi_0' \rangle = 1 + \langle G_0' \tilde{\pi} \Psi_0 | \psi_0' \rangle = 1 + \langle \psi_0' | t'^{+} G_0'^{+} | \psi_0' \rangle.$$

The error made in the normalization in computing $\langle \tilde{\pi} \rangle_0$ by (54) involves the operator combination of (53). This is to say that there has been a loss of probability from our H-F spectrum ψ_β. This has gone to make up the states ψ_β' reflecting the coupling to collective states through Δ_c. The change in the transition operator from $t(\pi_{01})$ to $t'(\tilde{\pi}_{01})$ reflects itself as a change of the two-particle self energy with total energy, at $E=E_0$.[10] Alternatively, there is a loss of normalization with respect to the original H-F basis set. The results of this paragraph imply an energy-shell approximation.

IV. Summary and Conclusions

We have chosen to discuss a reaction for which it is not possible to directly take over the measured 2-body scattering amplitude to evaluate that relating to complex systems. By working in the impulse approximation, we confront the usual energy shell problem. Apart from this, we have found it necessary to incorporate the random phase approximation into our reaction description. Such, introduces renormalization and hence further ambiguities in the relationship between measured 2-body amplitude and that, weighted by a form factor, which pertains to the interactions between complex systems.

The point of view expressed here has been that the elementary, 2-body, t-matrix is a dynamical operator. Associated with such an operator is an equation of motion. The specification of the latter must come from many-body theory. That theory, basically H-F theory restated in the formulation of Brueckner, is relatively unambiguous. Particularly, within its framework, it is possible to define what we mean by residual reactions in a satisfactory way. Such interactions are not always small. This is a fortunate and physically significant aspect for the finite nucleus. For, when we compute t in such a system, having its own degrees of freedom, the appearance of large residual interactions usually reflects the coupling of the single particle spectrum (in which t operates) to the col-

lective nuclear motions. While there are singularly few collective nuclear motions, lying at low frequency, which we can treat in a satisfactory quantitative way, the situation is far from being discouraging. Most often we couple the single particle spectrum to 2^+, 4^+ and 3^- electric vibrations. These couplings are quite easily extracted from the Brueckner cluster expansion. A partial summation procedure is used to do this.

In such a view as that just expressed, we give up any aim of describing the spatial distribution $\rho(\underset{\sim}{r})$ of the nuclear density; and thus also that of having $t(\rho)$ reflect the corresponding spatial behavior. For us, the finite nucleus is just a spectrum built on a particular class of states.

We consider the present treatment of the charge exchange problem to be closely related to that of (p,p') given by Levinson and Banerjee (L-B). Not so much emphasis was placed upon the H-F self-consistency by these authors. However, their work clearly implies and contains the optical potential aspect of self-consistency. This is chiefly expressed through the antisymmetrization in (A+1) nucleons and 2-nucleon dynamical equation. Note that, in this connection, we only have to antisymmetrize in the π-space of initial states, e.g., $2^{-1/2}\left[\psi_1(\pi_1)\,\psi_0(\pi_0) - \psi_1(\pi_0)\,\psi_0(\pi_1)\right]$, to make (31') self-consistent in this sense. But, actually, this is automatic owing to the way in which the charge exchange matrix element has been written down.

Both of the treatments, that of the present paper and (L-B), have much in common with that known as the resonating group procedure. (38) The latter contains the elements necessary for H-F self-consistency. Its chief difficulties lie in the unphysical approximations to the internucleon force and to certain wave functions made in order to: a) produce a two-particle dynamical equation, and; b) to resolve the kernel or integral operator which appears, over a suitable orthonormal basis. These aspects can in fact be avoided by the introduction of suitable t-approximations. Moreover, one has to exploit the fact that the effects owing to certain distorting interactions, e.g., optical potentials are in fact known. Strict attention to this fact was paid by (L-B) in their derivation of a two-particle dynamical equation. We have explained that the dynamical equation of this paper came from a parallel drawn from the Bethe-Goldstone equation.

In many ways the method of resonating group is superior in its presentation to that of the c.f.p. and cluster model as used here. This will continue to be true whenever complex nuclei, d,α,T, etc., are used as projectiles. At the other extreme, nucleon projectiles, an adaptation of the nuclear matter computations of Brueckner, Bethe and Shaw is preferred. Such an adaptation has as its basis the characterization of the finite nucleus according to techniques arising from field theory. Given an appropriate definition

of the physical vacuum, one goes ahead to construct the Green's functions G_1, G_2, and G_3. There are dynamical equations connecting the members of this hierarchy. Some approximations must be made in this scheme. However, these are non-perturbative and as such not generally damaging to the physics. In speaking of the Green's function hierarchy, we do not imply the use of ensemble Green's functions. Such are implied in the pairing theories, e.g., the work of Kisslinger and Sorensen. An example of the technique to which we allude here would appear in a computation of the hole-particle spectrum of C^{13}.

The field theory characterization emphasizes the role of correlations in the finite nucleus. Such can be induced or excited by nucleon (or complex nuclear) projectiles. The point is however that the correlations are not always reproduced by either the ladder or random-phase approximations. Also, contrary to the depiction of Brueckner and some of the work of this paper, particles do not always scatter from particles, nor holes from holes. Particles and holes should appear on the same footing. The Green's functions G_1, G_2, etc., are of the Feynman type and thus preserve this symmetry. Ferrell and his collaborators (39) have emphasized the role of correlations carried by the backward going graphs.

A collective excitation has been introduced in the present analysis. We computed in a simple-minded way how this excitation coupled into the H-F spectrum through an operator t. The description we gave was schematic. Its motivation lay in trying to reproduce a situation which was known to exist from an analysis of the reaction graphs. Of course, the physical nuclei involved always guide such an analysis. It is possible to sum the Brueckner cluster expansion using a Feynman projection operator off the chosen configuration to obtain the appropriate collective couplings. Some error is made in doing such a partial summation, since one does not choose to re-examine the Brueckner ladders, in order to correct them for redundant countings.

The introduction of the collective coupling leads us to expect certain renormalizations of the inter-nucleon force. This renormalization, and as well the "collective" vertex operator $\tilde{t}$, is quite strongly energy dependent, and A-dependent. It is then straight forward to envisage physical situations of more interest than that described here. Charge exchange, (p,n), for $E_p \approx 15$ Mev in the region of the Ni-isotopes will depend upon the processes described here. Low energy (He^3,T),E = 25 Mev, in this same region, will also be similarly governed by a similar description. High energy (He^3,T),$E \geq 40$ Mev introduces two additional considerations, namely strong absorption and the adiabatic

approximation. The microscopic, 2-body description given is not then particularly attractive. One instead resorts to discussions based upon considerations given by the Blair model.

A serious question may be raised having to do with our use of a two-state model. That was introduced to be able to include some aspect of H-F self-consistency. Moreover, such a model is formally tractable. To the point however is the observation that some account must be given of the virtual channels or states into which the actual reaction channels couple. This is a very old idea which was stressed by Thomas. (40)

We have been lead to distorted-wave expressions for our two-particle matrix elements. Again, such are only a reflection, here, of H-F self-consistency. This is one of the chief contentions of our analysis. It is possible to check the role of such an effect. The methods of this paper comprise a point of view. This stands alone as an exposition picture which derives from our qualitative understanding of reaction processes.

Machine computations are in progress for the example cited throughout, $Be^9(He^3,T)B^{9*}$ at 25 Mev. The computations are designed to compute -: 1) a plane wave approximation to the 2-body transition m.e., (31'); 2) the H-F statement of that m.e. with $\tilde{t}$, the collective coupling, set equal to zero; 3) finally, the m.e. of (31'). The results will be reported in Part II of this work.

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Footnotes

(1);(p.25) The c.m. system of the initial and final nuclei has been deliberately chosen for these energy arguments. This provides us with what is essentially a one-body equation of relative motion for the colliding nuclei. Obviously we satisfy the requirement of translational invariance. At the same time, it is consistent that elementary, two-body, matrix elements and wave functions be computed in some laboratory frame. That frame of reference is in motion with respect to our original one in which the system c.m. is at rest. The new frame is selected according to convenience, i.e., reaction model and associated wave functions. It is related to the original frame through a linear vector transformation carried out upon each particle coordinate.

(2);(p,26) This is a representative of a many-particle functional in which only the initially occupied states referring to "0" and "1" can be found empty. Initially occupied states are specified by the non-zero occupation numbers in a chosen configuration, taken here to have a two-particle representative Θ . The introduction of Θ and Ψ , as defined, allows us to solve an equivalent two-body Schrödinger equation. But, in a later section, III, we shall want to employ some results from many-particle (second-quantized) theories. At such a point, then, the functionals themselves are implied.

(3);(p.33) The wave function basis π_ν^J for two-hole, two-particle vibrations is discussed in Appendix B. Note the appearance of the time-reversed function $\tilde{\psi}$. If $j_1 = j$, the representation employed is that familiar from pairing theories. The choice of quantization axis $M_J = 0$ corresponds to viewing the nuclear excitation as resulting from nucleon interactions with an incident phonon, furnishing the quantization axis, of positive parity, multipolarity J. The arguments of ϵ_i, namely p and h, refer to particle and hole, respectively. The nomenclature quasi-particle (state) will in general refer to an excitation generated by taking linear combinations of shell model hole and shell model particle states.

(4);(p.34) The 2^+ state, here, occurs at very low frequency. Consequently, deformed shell model orbitals are implied. These are generated by introducing a one-body quadrupole operator into the H-F energy operator.

(5);(p.35) The details of this estimate are to be found in Appendix C. Underlying the present discussion is the notion that the 10.7 MeV B^{10} state may be represented as a composite of 2^+, one-phonon excitation plus single-particle excitations. The excitations envisaged, of the latter type, are that either the extra proton or extra neutron gets excited from $1p_{3/2}$ to $1p_{1/2}$. The binding

energy of the last nucleon in B^{10} is about 8.5 MeV. Clearly, these statements take on a somewhat different character when viewed from the $p+Be^{9}$ system; thus, the two-particle, one-phonon coupling scheme is introduced.

(6);(p.49) The discussion to follow can be equally well given in terms of (L-S) to $(j-j)$ transformation coefficients. The projection operator method used, was thought to be more compact and convenient in the present connection.

(7);(p.49) States of the added nucleon are denoted as k,k', etc. Those for the target nucleon appear here as μ but, however, generally involve a change of j owing to the interaction. The notation is then sufficiently general for the representation of the two-particle matrix elements.

(8);(p.58) The number quoted here, 19 Mev, is obtained as follows: $E_{k_0} = 23$ Mev, $E_{k'} = 35.2$ Mev, $\epsilon_{\mu_1} - \epsilon_{\mu_0}$ is the $1p_{1/2} - 1p_{3/2}$ splitting taken as 7.5 Mev; $E_{k'} + \epsilon_{\mu_1} - E_{k_0} - \epsilon_{\mu_0}$ = 19 Mev.

(9);(p.72) The operator for the energy shift, namely Δ_c, arising from interactions between independent excitations, e.g., quasi-particles, is introduced. As previously shown, the interactions come about through the exchange of the virtual phonons for collective excitations. We

shall then be led to an expression for the self-energy, or its change, owing to the coupling between quasi-particles and phonons.

(10);(p.74) Tne Brueckner ladder giving M* = 0.6 M corresponds to our $t(\pi_{01})$. On tne other hand $t'(\tilde{\pi}_{01})$ operating within the basis ψ'_β has been obtained by additional diagonalizations, or partial summations, yielding the couplings to collective motions. It is clear that M* so obtained will not in general equal that found for infinite matter. Tne latter supports compressional modes of wide variety, which are collective extensional and dilational disturbances. These are not under discussion here.

Appendix A.

The results of Section III depend upon certain aspects of the Brueckner theory. This is in particular true for Eqs. (26) and (27) and that following for the matrix element $\langle 0|\hat{v}|0\rangle$ where two successive t-interactions, leading back to the chosen configuration, are dropped. What we can employ of the Brueckner theory is the method. No single K-matrix element, e.g., that appearing in t_{11}^{-av}, computed by that theory can be used here.

The summation of graphs implies certain diagonalizations of the two-nucleon force. This in turn says that we know not only certain diagonal matrix elements but also the basis within which the diagonalization is carried out. Every diagonalization introduces new eigenvectors. To say that we want the t^2-terms out of the perturbation expansion for the ground state energy of the free-bound system is stringent. This means that the single-particle energies E_k and ϵ_μ have to be computed in such a manner that this is true. The oscillator well in which all of the particles are assumed to move gives a basis set. This well may even be assumed to be the H-F consistent potential. However, nothing has been done in the direction of the Brueckner method.

What is involved here is to determine t_{11}^{-av} for the target or A-particle system and then t_{00}^{-av} for scattering from the same system. The former is handled by writing

$$K = v + v\,\frac{Q}{e}\,K \qquad (a.1)$$

with

$$e = E_{\mu_0}(i) + E_{\mu_0}(j) - E'_{\mu}(i\,j\,;\mu_0) - E'_{\mu}(j\,i\,;\mu_0)$$

$$= E_0(n_i^0) + E_0(n_j^0) - E_1(n_i\,;n_i^0\,n_j^0) - E(n_j\,;n_i^0\,n_j^0). \qquad (a.2)$$

The symbols μ_0 and μ stand for the ground state target, or chosen, configuration and an excited configuration, respectively. In order to write the second of (a.2) an approximation owing to Brueckner has been used. It says that we may compute the energies of particles i and j which occupy states n_i and n_j in the excited configuration μ as a sum of effective single particle energies. In these energies, the influence of one member of the excited pair upon the other is taken into account in some average way. The single particle energies appearing above are written as

$$E_0(n_i^0) = \sum_{n_k^0}(n_i^0 n_k^0 \,|\, K_{ik}^0 \,|\, n_i^0 n_k^0)_A + (n_i^0 \,|\, T \,|\, n_i^0) = (n_i^0 \,|\, \bar{t}_{ii}^{av} + T \,|\, n^0)\,; \qquad (a.3)$$

$$E_1(n_i\,;n_i^0\,n_j^0) = \sum_{n_\ell^0}(n_\ell^0\, n_i \,|\, K_{i\ell} \,|\, n_\ell^0 n_i)_A + (n_i \,|\, T \,|\, n_i) = (n_i \,|\, t_{ii} + T \,|\, n_i)$$

The effective one-body operators t_{kk} are hermitian. This notation is introduced to correspond with that used earlier. There are problems of detail concerning how the operator K_{ij} differs from that K_{ij}^0. These are widely known and shall not be discussed here.

A wave-matrix statement is introduced, which defines what we mean by K°_{ij} ; this is

$$K^{\circ}_{i\jmath}\left\|\phi_n(i)\phi_m(\jmath)\right\| \equiv K^{\circ}_{i\jmath}\,\Theta_{\mu_0}(i\jmath) = v_{i\jmath}\,\Psi_{\mu_0}\,; \qquad (a.4)$$
$$(n = n^{\circ}_i,\ m = n^{\circ}_\jmath\ ;\ n^{\circ}_i, n^{\circ}_\jmath, n\,\mu_0)$$

The configuration μ_0 is the chosen one; Ψ_{μ_0} is the stationary state of interaction generated from interactions within μ_0. We have from (a.4) and (a.1) the integral equation

$$\Psi_{\mu_0}(i\jmath) = \Theta_{\mu_0}(i\jmath) + G_{\mu_0}\,v_{i\jmath}\,\Psi_{\mu_0}(i\jmath)\,;$$
$$G_{\mu_0}(i\jmath) = \sum_{\mu} |\,\Phi_\mu\rangle \frac{Q(\mu;\mu_0)}{E_{\mu_0} - E_\mu(i\jmath;\mu_0)} \langle\,\Phi_\mu\,|\,. \qquad (a.5)$$

And, if we can define the inverse of the Green's function as

$$G^{-1}_{\mu_0} = E_{\mu_0}(i\jmath) - \hbar(i) - h(\jmath)$$

then, the following differential equation is implied:

$$\left[E_{\mu_0} - \hbar(i) - h(\jmath)\right]\Psi_{\mu_0}(i\jmath) = Q_{\mu_0}\,v_{i\jmath}\,\Psi_{\mu_0}(i\jmath).$$

The one-body operators appearing here are those of (a.3). These we shall describe as the H-F operators. The stationary state will contain all configurations in two-particles which can be generated from μ_0. It is sufficient that we retain this information in the operator Q_{μ_0}. The two particle equation is rewritten as

$$\left[E(i\jmath) - \hbar(i) - \hbar(\jmath)\right]\Psi(i\jmath) = Q_{\mu_0}\,v_{i\jmath}\,\Psi(i\jmath). \qquad (a.6)$$

This type of equation was first written down by Bethe and Golastone. In solving the equation, we take into account H-F self-consistency and ignore that of Brueckner. To do this, we write

$$\bar{t}_{ii}^{av} = -1/2\, k\lambda_i^2 + a_{ls}\, \underset{\sim}{l}_i \cdot \underset{\sim}{s}_i \; ; \text{ and}$$

$$T_i = -(\hbar^2/2M^*)\, \nabla_i^2 . \qquad (a.1)$$

The first of the statements permits us to use oscillator wave functions. The second tells us that the oscillator spacing is $\hbar\omega = \hbar^2/M^*b$. We have introduced M* as an effective mass but this cannot be taken literally; b^2 is the oscillator parameter.

The approximation procedure consists of introducing a well and the wave functions $\Phi_\mu(ij)$ parametrically dependent upon the well. The stationary state is expanded as

$$\Psi(i\,j) = \sum C_\mu\, \Phi_\mu$$

This, when substituted into (a.6), leads to a secular equation. The unperturbed energies E(i), E(j) are generated from (a.7). It is assumed that v_{ij} is a regular potential. Solution of the secular equation yields a spectrum in energy E(ij). Taking the lowest value which appears here and the corresponding $\{C_{\mu_o}\}$ we then find $\Psi_{\mu_o}(ij)$. The K-matrix elements are computed with this function according to (a.4), i.e.,

$$(\Theta_{\mu_o} | K^o_{ij} | \Theta_{\mu_o}) = (\Theta_{\mu_o} | v_{ij} | \Psi_{\mu_o}) . \qquad (a.8)$$

According to this we have, <u>in the ground state</u>,

$$(n_i^o | \bar{t}_{ii}^{av} | n_i^o) = \sum_{n_j^o} (\Theta_{\mu_o} | K^o_{ij} | \Theta_{\mu_o})_A , \qquad (a.9)$$

as the interaction energy felt by the nucleon in state n . This will in general be different from the matrix element of $\bar{t}^{av}_{ii}$, as given by (a.7). Iteration of (a.7) is carried out until the two results agree. The result of this procedure is to determine the H-F consistent potential $\bar{t}^{av}_{ii}$. The K-matrix elements which emerge are not those computed from Brueckner tneory.

It is easy to see, nevertheless, that we have kept some of tne aspects of that theory. In particular from (a.3) and (a.5) it is evident that we have built into the H-F energy: all of the forward scatterings off of unexcited particles, and the sequence of self-energy insertions to infinite order in v_{ij}. It follows from these remarks that t^2-terms leading back to μ_o, the chosen configuration, do not exist.

Having completed the self-consistency computation for what we called $\bar{t}^{av}_{II}$ earlier, the interaction relating to the bound constituent, it is indicated that the nuclear binding energy be checked and also the r.m.s. nuclear radius. If both of these quantities come out correctly, we have been fortunate. It will not be surprising if the energy/particle comes out correctly but at too high a nuclear density. This has to do with our omission of collective excitations in the bound state problem. These permit a strong correlation structure in the presence of reduced nuclear densities.

To go to the Bethe-Goldstone equation for the free-bound system, namely to compute $\bar{t}^{av}_{oo}$ a modification of the procedure described is introduced. Given that we know $\bar{t}^{av}_{11}$, the corresponding H-F operator $\hbar(1)$ is specified. If the two-particle equation is written as

$$\left[E_{o1} - \hbar(0) - \hbar(1)\right] \Psi(0,1) = Q_{\nu_o} \upsilon_{o_1} \Psi(0,1) \qquad (a.10)$$

we must fix $\bar{t}^{av}_{oo}$ at this step. We have talked about particles "0" and "1" moving in a "common well". The added nucleon was described by a wave function $\eta_o^{(+)}(\hat{k}_o; \hat{r}_o)$ in the A + 1 particle chosen configuration. Clearly, the two nucleons move in a common well if they help determine the A + 1 average H-F potential for each other. We cannot solve this problem rigorously. However, a suitable approximation is available. An extra nucleon, j, is added to the A-particle target system. It has initially the $(\bar{t}^{av}_{jj})\mu_o$ interaction with the target, and is added in the A-particle H-F state lying closest to that of "0". The latter occupies the state of energy $\bar{E}_{k_o} = E_{k_o} - \epsilon_F = T(k_o) + B$; ϵ_F is the Fermi energy for the target, $T(k_o)$ is the kinetic energy of particle "0", $B > 0$ is the binding of the last nucleon in the ground state target. The dummy nucleon "j" and the ground state system form a chosen configuration λ_o in A+1 particles. The self-consistent H-F interaction $(\bar{t}^{av}_{jj})_{\lambda_o}$ can be computed by procedures already described, since j moves in the well of (a.7). Then for any well, $\bar{t}^{av}_{oo}$, e.g.,

$$\bar{t}^{av}_{oo} = -V\left(1 + a^{o}_{ls}\,\underset{\sim}{l}_{o}\cdot\underset{\sim}{s}_{o}\right) \quad r_{o} < R_{o}$$
$$= 0 \quad r_{o} > R_{o} \qquad \text{(a.11)}$$

the self-consistency will be considered to hold, if, $\bar{t}^{av}_{jj} = \bar{t}^{av}_{oo}$, in the sense that,

$$\left(\eta^{(-)}_{o}(\underset{\sim}{k}_{o};\underset{\sim}{r}_{o})\,|\,\bar{t}^{av}_{oo}\,|\,\eta^{(+)}_{o}(\underset{\sim}{k}_{o};\underset{\sim}{r}_{o})\right)_{\nu_{o}} = \left(\eta^{o}_{j}\,|\,\bar{t}^{av}_{jj}\,|\,\eta^{o}_{j}\right)_{\lambda_{o}}. \qquad \text{(a.12)}$$

The expression for the matrix elements of $\left(\bar{t}^{av}_{jj}\right)_{\lambda}$ is constructed from equations similar to (a.8) and (a.9). It is a restriction of this method that the states $n^{o}_{j}(\nu_{o})$ and $n^{o}_{j}(\lambda_{o})$ must not be separated in energy by more than the half-width of the optical potential state. This width is given in terms of W, the imaginary part of the optical potential. It cannot be obtained from the hermitian approximation employed here.

Appendix B.

The material of this Appendix was referred to at an earlier stage in the text than that given in Appendix A. However, the latter is helpful for the understanding of certain ideas and notations here.

The two-particle, two-hole unperturbed basis set Π_ν^J (1,2) is fairly complicated. Indeed, we should write for these objects

$$\Pi_\nu^J(1,2) = \frac{1}{\sqrt{2}} \sum_{l_i m_j = 1}^{2} \sum_{\mu_j = -m_j}^{m_j} \sum_{\lambda_i = -l_i}^{l_i} \begin{bmatrix} m_j & l_i \\ \mu_j & \lambda_i \end{bmatrix} \begin{vmatrix} j \\ m \end{vmatrix} \mathrm{Det} \| \psi_{l_i \lambda_i}(1) \tilde{\psi}_{m_j \lambda_j}(2) \| ; \quad (b.1)$$

the states l_i ($i = E_a, E_b \leq \epsilon_F$) are occupied, and those m_j ($j = E_k, E_p > \epsilon_F$) are unoccupied. The determinantal function is implied as normalized to unity. It can be demonstrated that the factor $2^{-1/2}$ leads to $\langle \nu J \mid \nu J \rangle = 1$. In order that this be established easily we have introduced symbols j, m. It is then the understanding that $\delta_{jJ}\,\delta_{m0}$ is implied in (b.1). The symbol ν stands, as before, for the excitation energy $E_k + E_p - E_a - E_b$ or a sum of two hole-particle energies.

It is assumed that the Π_ν^J diagonalize the energy operator $H_{12}^0 = h(1) + h(2)$, a sum of H-F operators. These are defined again in the sense of Brueckner. At this point such a definition is of considerable utility. This is to say that the perturbed, two-hole, two-particle eigenfunction Ψ^J can be taken as diagonalizing the operator $H_{12}^0 + v_{pp} + v_{ph} + v_{hh}$, or

$$(E - H^{o}_{12})\Psi^{J} = (v_{pp} + v_{ph} + v_{hh})\Psi^{J} \qquad (b.2)$$

The interactions (particle-particle, particle-hole, and hole-particle) appearing on the right correspond to the various factor-pairings of the two-body interaction in a second quantized, or many-particle, description. It is our understanding that the interaction are residual, i.e., not accounted for in H^{o}_{12} . As long as we restrict v_{pp} to arise from the cluster terms, it is non-vanishing. We have not said how the two-hole, two-particle states were established initially. It is within the context of our charge-exchange problem to argue that the B^{10*} states are generated by adding a proton to the Be^{9} ground state. Here, then, certain 3-body clusters appear. The B^{10*} states which we consider are to be enumerated in a manner consistent with these cluster contributions. It can then be argued that v_{pp} is small but non-zero. Technical problems arise in the counting of states above the Fermi energy ϵ_{F} . These involve details but no new physical considerations.

The hole-hole interaction v_{hh} is certainly not zero within a Brueckner formalism. It is sufficient to recall that hole states have zero widths within the formalism. One must also recall that finite nuclei of intermediate and low mass number may not resemble infinite nuclear matter with its characteristically large excitation energies. The particle-hole interaction v_{ph} is clearly non-zero

within the formalism of Brueckner theory. It represents the largest undiagonalized contribution to the interaction. In view of this it is easily understood why the vibration Ψ^J is often represented as a pair of hole-particle vibrations, v_{pp} and v_{hh} being taken equal to zero in this approximation.

Eq. (b.2) is to be solved by making the substitution $\Psi^J = \sum c_\nu \Pi^J_\nu$ providing us with the usual secular equation. It might be remarked that v_{pp} is the easiest of the residual interactions to estimate within the formalism we have chosen. That, v_{hh} , can be gotten at through certain experimental evidence such as (p,2p). It probably does even in finite nuclei involve considerable excitations. The force v_{ph} is the subject of much current investigation. The preliminary conclusions seem to indicate that it is nearly pure exchange force and, capable of giving saturation. In view of our restrictions to residual forces, the latter presents a clear difficulty.

Appendix C.

This section contains an estimate of the spin-orbit splitting $p_{1/2}-p_{3/2}$ in the A=10 system, $n+Be^9$. The nuclear system is formed in elastic neutron scattering from Be^9. We expect to reach states of excitation 6.82 Mev or greater in the scattering of thermal and slow neutrons from Be^9. The Be^9 system is so light that for the bombarding energies ($0.3 < E_n < 1$ Mev) of interest we shall be in a Breit-Wigner region. Scattering resonances instead of those for capture are expected here. In order to pass to a single particle picture, that of the optical potential, the A=10 level densities should be large. This condition begins to be satisfied at about E_n = 1 Mev. We shall for the purpose of simplicity assume that there are enough levels to average over - that the optical potential applies. The error made will not be too large, if the splitting corresponds to a value of $E_n \geq$ 0.5 Mev. Also, at such energies, states J^π, equal to $1^+, 2^+$ are among those formed in the reaction. The method of analysis used is only satisfactory for the location of the 1^+ state. In the body of this paper it is considered that a collective 2^+ state is excited in B^{10} owing to the addition of a proton to Be^9. The position of this state is determined by that of the 1^+,T=1 state, under discussion, together with the Be^{8*} phonon excitation of 2.9 Mev. The arguments to follow are restricted to the inference of the single particle splitting $E(1/2^-)-\epsilon(1p_{3/2})$.

The method of computation here is based upon a perturbation expansion of the two-particle interaction energy, v . In the first two orders of perturbation theory, the single particle energy of a neutron added in the $1p_{1/2}$ state to the A=9 ground state, will be $E(1/2^-)+V_I(1/2^-)$ with $V_I(1/2^-)$ being given by the diagrams of Figs. 1C(a) - 1C(c). The parameters of the interaction v are fixed by computing the binding energy per particle and separation energy of the last nucleon in the A=9 (Be^9 ground state) system. This involves determination of the H-F potential and the strength, α, of the spin-orbit interaction for the A=9 nucleons. No attempt is made to achieve H-F self-consistency. The computation is first of all illustrative and, next, should contain a proper two-body force, a realistic two-body reaction matrix, to make H-F self-consistency worth-while. A Serber force of zero range is used for the two-particle interaction v . It is convenient to work within the basis of oscillator functions. The size parameter for these is fixed from the mean square, A=9, radius. We find that it is consistent to explain the low binding energy of the last A=9 nucleon as arising from a renormalization, x, of the two-nucleon force. Such comes about if this nucleon moves in the presence of collective, A=8, core or vacuum fluctuations. Both the volume strength, λ, of the two-nucleon force and the renormalization constant x are determined.

A finite well is implied throughout, both for nucleon-nucleon and nucleon-nucleus interactions. The configurations relevant to the latter are depicted in Fig. 2C. The position of the state $E(1/2^-)$ is fixed by requiring that the real part of the optical potential, Re $\mathcal{V}_{op}(1/2^-)$, equal Re $V_I(1/2^-)$. From Fig. 1C, we see that the diagrams involve $E(1/2^-)$ and thus the equality gives an algebraic equation for $E(1/2^-)$.

The appropriate relations for the binding energy per particle and the mean square nuclear radius using oscillator functions are

$$r_o^2 = \frac{\sum \langle r^2\rangle_{n\ell} W_{n\ell}}{\sum W_{n\ell}}, \quad \langle E\rangle/N = \frac{\sum \epsilon_{n\ell} W_{n\ell}}{\sum W_{n\ell}};$$
$$\langle r^2\rangle_{n\ell} = \lambda\!\!\!^{-2}_c \left(Mc^2/\hbar\omega\right)\left(\Lambda_{n\ell}+3/2\right), \; \epsilon_{n\ell} = \hbar\omega\left(\Lambda_{n\ell}+3/2\right), \; \sum W_{n\ell} = N \qquad \text{(c.1}$$

Here, N is the total number of nucleons; $W_{n\ell}$ is the occupation of the oscillator shell, equal to $2\cdot(\Lambda_{n\ell}+1)(\Lambda_{n\ell}+2)$ for a filled shell; $\Lambda_{n\ell} = (2n+\ell-2)$ where n is the principal quantum number and ℓ is the orbital angular momentum; $\lambda\!\!\!^{-}_c$ is the nucleon Compton wave length; Mc^2 = 938.2 Mev; $\hbar\omega$ is the oscillator spacing equal to $\hbar^2/Mb^2$, with the size parameter b being given in terms of the force parameter K as $b^{-4} = MK/\hbar^2$. On the assumption of a uniform charge distribution, the second moment of the distribution is $r_o^2 = (0.6)(1.25A^{1/3})^2 f^2$. For the A=9 system, the value $\hbar\omega$ equal to 20.8 Mev agrees well with $\hbar\omega = 19.7$ from $41/A^{1/3}$ applying

to saturated (normal density) systems. These details are repeated for completeness and reflect one way in which one may treat the electron scattering data. We have now defined a set of independent particle wave functions.

The two-nucleon potential is taken to be

$$v_{12}(r) = v^{(1)} + \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2 v^{(2)}, \quad v^{(1)} = \frac{1+P^M}{2} \lambda_i \, \delta(\underset{\sim}{r}_1 - \underset{\sim}{r}_2); \qquad \text{(c.2)}$$

a Serber force of zero range. We take the ratio of singlet to triplet volume strengths as equal to 0.6, whereupon $\lambda_2 = 0.11 \lambda_1$. The nature of the assumed force has relevance for the form in which the wave functions are presented. In computing the H-F potential, we encounter matrix elements of the form

$$\left(\phi_1(1)\, \phi_2(2) \right| v_{12} \left| \begin{vmatrix} \phi_1(1) & \phi_1(2) \\ \phi_2(1) & \phi_2(2) \end{vmatrix} \right).$$

The Slater determinant is not to be normalized and the single particle functions ϕ are here taken as products of $\ell - \ell$ basis functions $\phi_{n\ell\gamma\mu}$ and those in isotopic spin $\xi^t_{1/2}$. A Serber force allows us to write the determinantal function as

$$1/2 \left(a^1_{\ell_1 m_1} a^2_{\ell_2 m_2} + a^2_{\ell_1 m_1} a^1_{\ell_2 m_2} \right) \begin{vmatrix} a^1_{s_1 t_1} & a^2_{s_1 t_1} \\ a^1_{s_2 t_2} & a^2_{s_2 t_2} \end{vmatrix}.$$

The determinant is now in the space of charge-spin. All of the information as to $\ell - \ell$ coupling basis is contained in the missing, but implied, vector coupling (v.c.) co-

efficients. Throughout the analysis, a charge-independent force is employed. Moreover, the dependence of the matrix elements of v_{12}, Eq. (c.2), upon isotopic spin states is completely trivial. We perform an average over the i-spin states to obtain

$$(v_{12})_{m.e.} = \left[\langle \alpha^1_{s_1'} \alpha^2_{s_2'} | v_{12} | \alpha^1_{s_1} \alpha^2_{s_2}\rangle - \frac{1}{2}\langle \alpha^1_{s_1'} \alpha^2_{s_2'} | v_{12} | \alpha^2_{s_1} \alpha^1_{s_2}\rangle\right] E(\ell_1 m_1 \ell_2 m_2; 12). \quad (c.3)$$

This result says that the charge symmetric force is written as $[(1/4)(V_{nn}+V_{pp})+(1/2)V_{np}] = (1/2)(V'+V_{np})$. Then the charge independent force is $V = V' = V_{np}$. The factor E(12) in Eq. (c.3) represents the space part of the v_{12}-matrix element.

We shall give two forms for the matrix element of v_{12}. The first of these is in the representation $\alpha=(n\ell j\mu)$. The second form is that for the representation $\beta=(n\ell j)$, namely the <u>average filled shell potential</u>. In computing the H-F potential from the β-matrix elements, one multiplies the diagonal, two-particle matrix element by the occupation factor $w_{n\ell j}$. The α-matrix elements depend upon projection, μ. To use these for incomplete shells, more possible states than particles can mean, in the simplest applications, that coupling rules, e.g., Mayer-Jensen, $j-j$ coupling, are required. However, the square, potential matrix in occupied states, from which the H-F potential is obtained, may be constructed in terms of operators projecting off the empty states. To such a construction, carried

out by enumeration, we attach the isotopic spin labels through V_{nn} , V_{np} , V_{pp} . Then, independent of coupling rules, it is possible to obtain the H-F potential for a partially filled shell. In the α-representation, we have the matrix elements

$$v^{(1)\,d\lambda} = -(1/2)(-1)^{\mu_1+\mu_2+\ell_1+\ell_2}\left[\gamma_1\right]\left[\gamma_2\right]\left(\left[\ell_1\right]\left[\ell_2\right]\right)^{1/2}\sum_L \begin{bmatrix} \gamma_1 & \gamma_1 & L \\ \mu_1 & -\mu_1 & 0 \end{bmatrix}\begin{bmatrix} \gamma_2 & \gamma_2 & L \\ \mu_2 & -\mu_2 & 0 \end{bmatrix}$$
$$\left[L\right]^{-1} g_L^{(1)}(n_1\ell_1^2\, n_2\ell_2^2)\, W(\gamma_1\gamma_1\ell_1\ell_1; L\tfrac{1}{2})\, W(\gamma_2\gamma_2\ell_2\ell_2; L\tfrac{1}{2}) \qquad \text{(c.4a)}$$

$$v^{(1)\,ex.} = -(1/2)(-1)^{\ell_1+\ell_2}\left[\gamma_1\right]\left[\gamma_2\right]\sum_{f\lambda}\left[W(\gamma_2\ell_2\gamma_1\ell_1; \tfrac{1}{2}f)\right]^2\begin{bmatrix} \gamma_2 & \gamma_1 & f \\ -\mu_2 & \mu_1 & -\lambda \end{bmatrix}^2(-1)^{-f}$$
$$\sum_L (-1)^L q_L^{(1)}(n_1\ell_1^2\, n_2\ell_2^2)\, W(\ell_1\ell_1\ell_2\ell_2; Lf) \qquad \text{(c.4b)}$$

$$v^{(2)\,d\nu} = 2(-1)^{\gamma_1+\gamma_2}\left[\gamma_1\right]\left[\gamma_2\right]\left[1\right]\sum_L q_L^{(2)}(n_1\ell_1^2\, n_2\ell_2^2)$$
$$\sum_f (-1)^{-f}\left[f\right] W(\gamma_1\tfrac{1}{2}L\ell_1; \ell_1 f) W(\gamma_1\ell_1 1\tfrac{1}{2}; \tfrac{1}{2}f)\sum (-1)^{-q}\left[q\right] W(\gamma_2\tfrac{1}{2}L\ell_2; \ell_2 q) W(\gamma_2\ell_2 1\tfrac{1}{2}; \tfrac{1}{2}q)$$
$$\sum_\nu \begin{bmatrix} \gamma_1 & \gamma_2 & \nu \\ -\mu_1 & -\mu_2 & -\rho \end{bmatrix}^2 W(\gamma_1 f \gamma_2 q; 1\nu)\, W(\gamma_1 f \gamma_2 q; L\nu) \qquad \text{(c.4c)}$$

$$v^{(2)\,ex.} = -\left[\gamma_1\right]\left[\gamma_2\right]\left[1\right](-1)^{\gamma_2-\gamma_1}(-1)^{\ell_1+\ell_2+\gamma_1+\gamma_2}\sum_L (-1)^L q_L^{(2)}(n_1\ell_1^2\, n_2\ell_2^2)$$
$$\sum_{\nu f}(-1)^f\left[f\right] W(\ell_1\ell_1\ell_2\ell_2; Lf)\begin{bmatrix} \gamma_1 & \gamma_2 & \nu \\ \mu_1 & -\mu_2 & \rho \end{bmatrix}^2\begin{Bmatrix} \tfrac{1}{2} & \tfrac{1}{2} & 1 \\ \ell_1 & \ell_2 & f \\ \gamma_1 & \gamma_2 & \nu \end{Bmatrix}^2 \qquad \text{(c.4d)}$$

The corresponding matrix elements in the β-representation are

$$v^{(1)^{dh}} = -(1/2)\left([\ell_1][\ell_2]\right)^{1/2}(-1)^{\ell_1+\ell_2}\sum_L g_L^{(1)}(n_1\ell_1^2 n_2\ell_2^2)W(\lambda_1\lambda_1\ell_1\ell_1;L\,1/2)W(\lambda_2\lambda_2\ell_2\ell_2;L\,1/2)$$
$$\sum_f (-1)^{-f}[f]\,W(\lambda_2\lambda_2\lambda_1\lambda_1;Lf) \qquad \text{(c.5a)}$$

$$v^{(1)^{ex}} = -(1/2)(-1)^{\ell_1+\ell_2}\sum_L g_L^{(1)}(n_1\ell_1^2 n_2\ell_2^2)(-1)^L$$
$$\sum_f (-1)^{-f}[f]\left[W(\lambda_2\ell_2\lambda_1\ell_1;1/2\,f)\right]^2 W(\ell_1\ell_1\ell_2\ell_2;Lf) \qquad \text{(c.5b)}$$

$$v^{(2)^{dh}} = \sum_L F_L(n_1\ell_1\lambda_1;n_2\ell_2\lambda_2)\,\delta_{1L}=0 \qquad \text{(c.5c)}$$

$$v^{(2)^{ex}} = -[1](-1)^{\lambda_1-\lambda_2}(-1)^{\ell_1+\ell_2+\lambda_1+\lambda_2}\sum_L (-1)^L g_L^{(2)}(n_1\ell_1^2 n_2\ell_2^2)$$
$$\sum_f (-1)^{-f}[f]\,W(\ell_1\ell_1\ell_2\ell_2;Lf)\sum_h [h]\begin{Bmatrix} 1/2 & 1/2 & 1 \\ \ell_1 & \ell_2 & f \\ \lambda_1 & \lambda_2 & h \end{Bmatrix}^2 \qquad \text{(c.5d)}$$

The notation for the v.c. coefficient, Racah coefficient, and (9-λ) symbol is standard; $[k]$ means $(2k+1)$; $g_L^{(i)}(n_1\ell_1^2\ n_2\ell_2^2)$ is defined as, $(i=1,2)$,

$$g_L^{(i)}(n_1\ell_1^2 n_2\ell_2^2) = \lambda_i\, I(n_1\ell_1^2 n_2\ell_2^2)(\ell_1\|Y_L\|\ell_1)(\ell_2\|Y_L\|\ell_2);$$

$$I(n_1\ell_1^2\ n_2\ell_2^2) = \int dh\, h^2\, R^2_{n_1\ell_1}(h)\, R^2_{n_2\ell_2}(h),$$

in terms of the Slater integral of oscillator functions, the reduced matrix elements of the irreducible tensor operator for the potential multipole, and the potential strength. Although the answer is well known, the matrix element of the one-body spin-orbit force is appended to the preceding list of formulas.

$$V^{s.o.}(n_1 \ell_1 \jmath_1 \mu_1) = -\alpha \left(\ell_1 [\ell_1] (\ell_1+1)\right)^{1/2} \sqrt{3/2}\,(-1)^{\ell_1+\frac{1}{2}-\jmath_1-1}\, W\left(\tfrac{1}{2}\tfrac{1}{2}\ell_1\ell_1; 1\jmath_1\right) \qquad (c.6)$$

The parameter α is the multiplicative factor in the force written as $\alpha\, \underset{\sim}{\ell}\cdot\underset{\sim}{s}$.

We can at this point compute the H-F potential. This is done in the β-representation. Two equations are used to fix the values of λ_1, and α. These arise from the requirements that the average binding per nucleon, $B = E/N$, and the separation energy, S_n, for the last neutron be given correctly. In the A=9, Be^9 ground state the values of these quantities are B = -6.46 Mev, and S_n = -1.7 Mev. The Slater integrals which appear are: $I(1p^4) = (5/6)(b^{-3}/\sqrt{\pi})$; $I(1s^4) = 2(b^{-3}/\sqrt{\pi})$; $I(1s^2, 1p^2) = I(1p^2, 1s^2) = (1/\sqrt{2})(b^{-3}/\sqrt{\pi})$. Using the definition $e_1 = (\lambda_1/4\pi b^3)$, with λ_1 being measured in Mev·f^3, we obtain diagonal two-particle matrix elements with the following values: $(1p^2_{3/2} \mid v \mid 1p^2_{3/2}) = 0.593\ e_1$; $(1p_{3/2}\,1s_{1/2} \mid v \mid 1s_{1/2}\,1p_{3/2}) = 0.24\ e_1$; $(1s^2_{1/2} \mid v \mid 1s^2_{1/2}) = 0.508\ e_1$. The H-F potentials which we obtain from these values are, (A=9),

$$V_{H.F.}(1s_{1/2}) = 1.72\, e_1\ ,\ V_{H.F.}(1p_{3/2}) = 3.13\, e_1\ .$$

In the oscillator representation, $\hbar\omega$ = 19.7 Mev, the single particle kinetic energies are $T(1s_{1/2})$ = 14.8 Mev, $T(1p_{3/2})$ = 24.6 Mev. Introducing the required factors of (1/2) $V_{H.F.}$ to compute the energy sum, we find that B is given by

$$-6.46 = 20.24 - 1.25\, e_1 - 0.278\,\alpha. \qquad (c.7)$$

To obtain the second equation, that for S_n , we consider the last neutron to move in the field of a fluctuating A=8 core. This core undergoes collective oscillations, the maintenance of which is achieved only by depleting a part of the single particle probability distribution. With the latter there is also associated a loss of matrix element. This can be thought of as a reduction in the strength of the average nuclear field. The effect is state dependent. It is largest for the states at the top of the Fermi distribution. Conversely it is negligible for the states at the bottom of that distribution. To calculate the force renormalization requires the use of more complicated reaction matrix operators than have to date appeared in the literature. The expedient adopted here is to replace e_1 by xe_1 , $(x < 1)$, in the equation for S_n which then reads

$$-1.7 = 24.6 - 1.565\,x e_1 - 0.5\,\alpha. \qquad (c.8)$$

The two requirements: 1) that we cancel out the assumed binding field, the oscillator well, as the first and trivial step in achieving H-F self-consistency; 2) that the spin-orbit force lie in the neighborhood of values given by intermediate coupling calculations, then fix the parameters of Eqns. (c.7) and (c.8) as

$$x = 0.77, \quad a = 2.4 \text{ Mev}, \quad e_1 = 20.8 \text{ Mev}.$$

The mean square radius determination for A=9 gives a value of 1.48 f for b. Then, λ_1 equals 855 Mev·f^3.

To estimate the contributions arising from the graphs of Figs. 1C,b) and 1C,c), we adopt the following physical picture. The incoming neutron incident upon the ground state A=9 system is at sufficiently high energy ($k_n^0 \cdot \sqrt{\langle r_0^2 \rangle} = \ell_n \cdot \hbar$; $\ell_n = 1$) that both s- and p-state phase shifts are important. In its interaction with the target, the neutron de-excites to the s-state, $k_n = k = 0$. The target excitation in this intermediate state is restricted to be no greater than $\hbar\omega$. Subsequent interaction between target and neutron returns us to the initial state. This picture allows us to use effective range theory; an enormous simplification. Furthermore, the two possible vacuum suppression contributions ($\ell_1 < \epsilon_F$, $m > \epsilon_F$: $\ell_1 = (1p_{3/2}; S_n)$, $\ell_0 = (1p_{3/2}; B)$, $m=0$; $\ell_1 = (1p_{3/2}; S_n) = \ell_0$, $m=0$; $m=0$ being $E(k=0)$) vanish rigorously. We are left with, then, the self-energy term which is written as

$$V_{s.e} = \left|\left(\tfrac{1}{2}^+\tfrac{1}{2}^+|v|\tfrac{1}{2}^-\tfrac{3}{2}^-\right)_A\right|^2 \frac{13.26 - 5E(\tfrac{1}{2}^-)}{[1.7 - E(\tfrac{1}{2}^-)][6.46 - E(\tfrac{1}{2}^-)]} . \qquad (c.9)$$

Here, as in all of the preceding work, we compute anti-symmetrized matrix elements. The state '0' is that with zero kinetic energy, E(k=0), and spin-parity, $1/2^+$. Again, effective range theory gives the scattering wave function. It is important to point out the sum-over-intermediate states, which is implied, will be simulated by varying $E(1/2^-)$ in the equation $\mathrm{Re}\,\mathcal{V}_{op}(1/2^-) = \mathrm{Re}\,V_I(1/2^-)$. The matrix element appearing in Eq. (c.9) is computed as

$$\left|\left(\gamma\gamma|v|\gamma_1\gamma_2\right)_A\right|^2_{a.v.} = \frac{1}{[\gamma][\gamma_1][\gamma_2]} \sum \left|\left(\phi_\alpha^\nu(1)\phi_{\alpha'}^{\nu'}(2)|v|\phi_{\alpha_1}^{\mu_1}(1)\phi_{\alpha_2}^{\mu_2}(2)\right)_A\right|^2 \qquad (c.10)$$

which is the form appropriate to the β-representation. It is unfortunately too tedious to reproduce all of the formulas pertaining to Eq. (c.10). The most useful, and also those leading to the non-vanishing contributions in our final result, are given below for reference and aid to future computations. Note first that, in Eq. (c.10) the following definitions apply $\alpha_1 = 1p_{1/2}\mu_1$, $\alpha_2 = 1p_{3/2}\mu_2$, $\alpha = s_{1/2}\nu$, $\alpha' = s_{1/2}\nu'$ in the α-<u>representation</u>. Thus, we have $\ell_1 \equiv \ell_2$ and $\ell' \equiv \ell = 0$ in the following formulas.

$$v^{(1)^{dir}} = (1/2)\left([\gamma_1][\gamma_2]\right)^{1/2}[\ell]^{-1} g^{(1)}_{\ell_1}(o^2;\eta_1\ell_1^2) \sum_M \begin{bmatrix} \gamma_1\ \ell_1 \\ \mu_1\ M \end{bmatrix} \begin{bmatrix} \gamma \\ \nu \end{bmatrix} \begin{bmatrix} \gamma_2\ \ell_1 \\ \mu_2\ -M \end{bmatrix} \begin{bmatrix} \gamma \\ \nu' \end{bmatrix} (-1)^M \qquad (c.11a)$$

$$v^{(1)^{ex}} = -(1/4)\left([\gamma_1][\gamma_2]\right)^{1/2}[\ell_1]^{-2} g^{(1)}_{\ell_1}(o^2;\eta_1\ell_1^2) \sum_\lambda \begin{bmatrix} \gamma_1\ \ell_1 \\ \mu_1\ \lambda \end{bmatrix} \begin{bmatrix} \gamma \\ \nu' \end{bmatrix} \begin{bmatrix} \gamma_2\ \ell_1 \\ \mu_2\ -\lambda \end{bmatrix} \begin{bmatrix} \gamma \\ \nu \end{bmatrix} (-1)^\lambda \qquad (c.11b)$$

$$v^{(2)\,dir} = [1]([j_1][j_2])^{1/2}(-1)^{-2j-\nu-\nu'}(-1)^{j_1+j_2-\mu_1-\mu_2}(-1)^{l_1}[l_1]^{-1} g^{(2)}_{l_1}(o^2; n\,l^2)$$

$$\sum_{p\pi}(-1)^{-p-\pi}[p]\begin{bmatrix} j_1 & p \\ \mu_1 & -\pi \end{vmatrix} \begin{matrix} j \\ \nu \end{matrix}\end{bmatrix}\begin{bmatrix} j_2 & p \\ \mu_2 & \pi \end{vmatrix} \begin{matrix} j \\ \nu' \end{matrix}\end{bmatrix}\sum_f (-1)^f [f] W(j_1 l_1 1/2; 1/2 f) W(j l_1 j_1 1; f p)$$

$$\sum_q (-1)^{-q}[q] W(j_2 l_1 1/2; 1/2\, q) W(j l_1 j_2 1; q p) \qquad \text{(c.11c)}$$

$$v^{(2)\,ex} = (1/2)[1]([j_1][j_2])^{1/2}(-1)^{-\mu_1-\mu_2}(-1)^{l_1}[l_1]^{-1} g^{(2)}_{l_1}(o^2; n_1 l_1^2)$$

$$\sum_{p\pi}(-1)^{-p-\pi}\begin{bmatrix} j_2 & j_1 & | & p \\ \mu_2 & \mu_1 & | & -\pi \end{bmatrix}\begin{bmatrix} j & j & | & p \\ \nu & \nu' & | & -\pi \end{bmatrix}\sum_{qr}(-1)^{q+j_2+2r}[q][r] W(j_2 l_1 1/2; 1/2\, q)$$

$$W(j_1 l_1 1/2; 1/2\, r) W(j_2\, q\, j_1 r; 1 p) W(1/2\, l_1\, p r; q\, 1/2) \qquad \text{(c.11d)}$$

In the formulas to follow, we use D to mean direct and X to mean exchange. The formulas as given describe the second order transition of two nucleons in the orbital $n_1 l_1$ to the state of zero kinetic energy. This is true although we compute a self-energy for a particle and not a hole state.

$$|(1/2\,1/2|v|j_1 j_2)_A|^2_{av} = \frac{1}{[j_1][j_2]}\Big\{[l_1]^{-1}((D^{(1)}_{l_1})^2+(X^{(1)}_{l_1})^2)+\sum_p[p]((D^{(2)}_p)^2+(X^{(2)}_p)^2)$$

$$+ 2\, D^{(1)}_{l_1} X^{(1)}_{l_1}\sum_k(-1)^k[k][W(j_1 j_2\, 1/2\, 1/2; k l_1)]^2(-1)^{2j_1}$$

$$-(-1)^{j_1+j_2-l}[l_1]\, D^{(1)}_{l_1} D^{(2)}_{l_1}+(-1)^{2j_2+j_1-1/2} D^{(1)}_{l_1}\sum_s(-1)^s X^{(2)}_s W(1/2\, j_1, 1/2\, j_2; l_1 s) \qquad \text{(c.12a)}$$

$$-(1/2)(-1)^{1/2+j_2}\sum_{ps}[p](-1)^{-s}[s]\, D^{(2)}_p X^{(2)}_s W(1/2\, j_1\, 1/2\, j_2; p s)\Big\}$$

$$D^{(1)}_{\ell_1} = (1/2)\left([j_1][j_2]\right)^{1/2}[\ell_1]^{-1}\, q^{(1)}_{\ell_1}(0^2; n_1\ell_1^2) \qquad \text{(c.12b)}$$

$$X^{(1)}_{\ell_1} = -(1/4)\left([j_1][j_2]\right)^{1/2}[\ell_1]^{-2}\, q^{(1)}_{\ell_1}(0^2; n_1\ell_1^2) \qquad \text{(c.12c)}$$

$$D^{(2)}_{p} = [1]\left([j_1][j_2]\right)^{1/2}(-1)^{\ell_1}[\ell_1]^{-1}\, q^{(2)}_{\ell_1}(0^2; n_1\ell_1^2)\sum_f (-1)^{-f}[f]\, W(j_1\ell_1 1/2; 1/2\, f)\, W(1/2\,\ell_1 j_1 1; f\, p)$$
$$\sum_q (-1)^{-q}[q]\, W(j_2\ell_1 1/2; 1/2\, q)\, W(1/2\,\ell_1 j_2 1; q\, p) \qquad \text{(c.12d)}$$

$$X^{(2)}_{s} = (1/2)[1]\left([j_1][j_2]\right)^{1/2}(-1)^{\ell_1}[\ell_1]^{-1}\, q^{(2)}_{\ell_1}(0^2; n_1\ell_1^2)\sum_{qr}(-1)^{q+j_2+2r}[r][q]\, W(j_2\ell_1 1/2; 1/2\, q)$$
$$W(j_1\ell_1 1/2; 1/2\, q)\, W(j_2\, q\, j_1\, r; 1s)\, W(1/2\,\ell_1\, s\, r; q\, 1/2) \qquad \text{(c.12e)}$$

The factors $q^{(1)}_{\ell_1}(0^2; n_1\ell_1)$ are defined in an exactly analogous manner to that previously employed. Only this time, a different Slater integral will appear, namely

$$I(0^2; n_1\ell_1^2) = \int dr\, r^2\, \psi^2_{k=0}(r)\, R^2_{n_1\ell_1}(r). \qquad \text{(c.13)}$$

Our procedure for handling the scattering state has been to employ the <u>interior</u> wave function from effective range theory, i.e.

$$\psi_{k=0}(r) = \Omega^{-1/2}\left(1 - r/a_s\right). \qquad \text{(c.14)}$$

The scattering length, a_s , is taken to be equal to the range of the nuclear potential, r_0 of Eq. (c.1). Also, the quantization volume Ω is taken equal to the A=9 nuclear volume. The approximation for a_s leads to an underestimate of $I(0^2;n,l_1^2)$. A further diminishment occurs if the integral is cut off at $r=r_0$. To obtain a roughly compensating situation, the integral was extended to infinity. The value so obtained, namely $I(0^2;11^2) = 7.9 \times 10^{-3}$, was however checked against that, 5.1×10^{-3}, obtaining from numerical integration of the integral with a cut-off.

We still require the one-body potential for the neutron in the $1p_{1/2}$ state. In this potential, there is a contribution from the repulsive spin-orbit force. Of course, all of the interaction strengths have been fixed by the considerations applying to the A=9 system. We find now that

$$V(1/2^-) = V_{H.F.}(1p_{1/2}) - \alpha = 2.813\, e_1 - \alpha = -56.1 \text{ Mev.} \qquad (c.15)$$

The final equation determining $E(1/2^-)$ is, then, using

$$\text{Re}\ v_{op}(1/2^-) = -45 \text{ Mev} + 0.5\, E(1/2^-),$$

$$11.1 + 0.5\, E(1/2^-) = \frac{13.26 - 5\, E(1/2^-)}{\left[1.7 - E(1/2^-)\right]\left[6.46 - E(1/2^-)\right]} 6.95 \times 10^{-2}. \qquad (c.16)$$

The solution of this is $E(1/2^-) = 6.4$ MeV, or a spin-orbit splitting $1p_{1/2}-1p_{3/2}$ of $E(1/2^-)+1.7 = 8.1$ MeV. We believe that we overestimate the splitting by perhaps 0.5 MeV due to improper treatment of the principal value. Otherwise, the technique is certainly useful. With the advent of better reaction matrices, a more careful computation would be warranted.

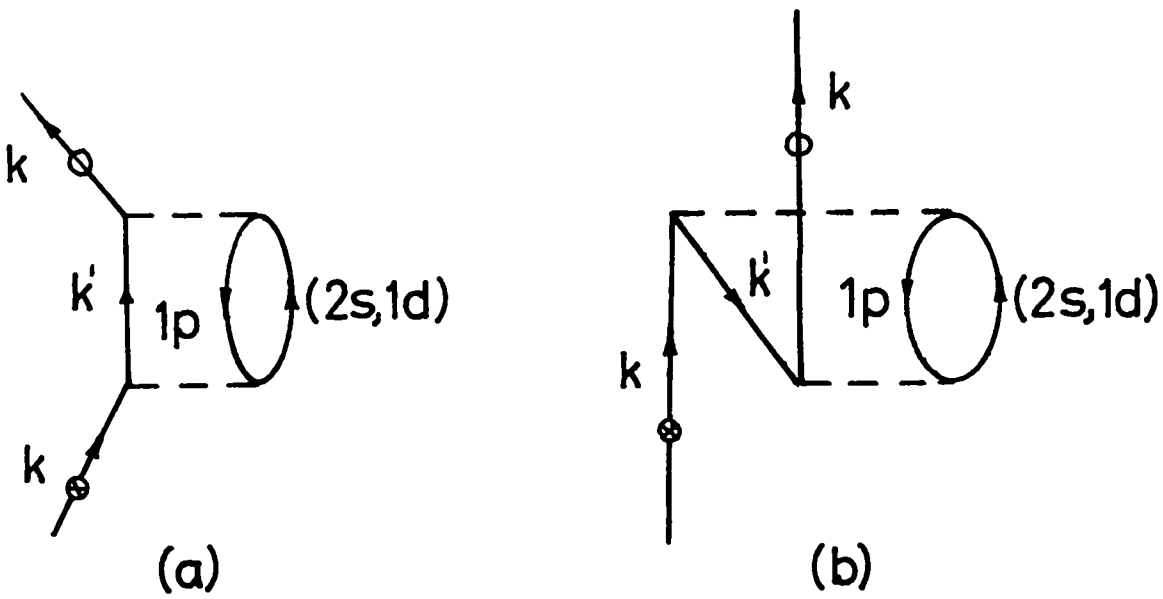

Fig. 1. Potential interactions, v, leading to vanishing energy denominators in second order. The open circle, o, on an external line refers to neutron; that, ⊗ refers to a proton. Coulomb forces are neglected. H-F potentials appear in first order.

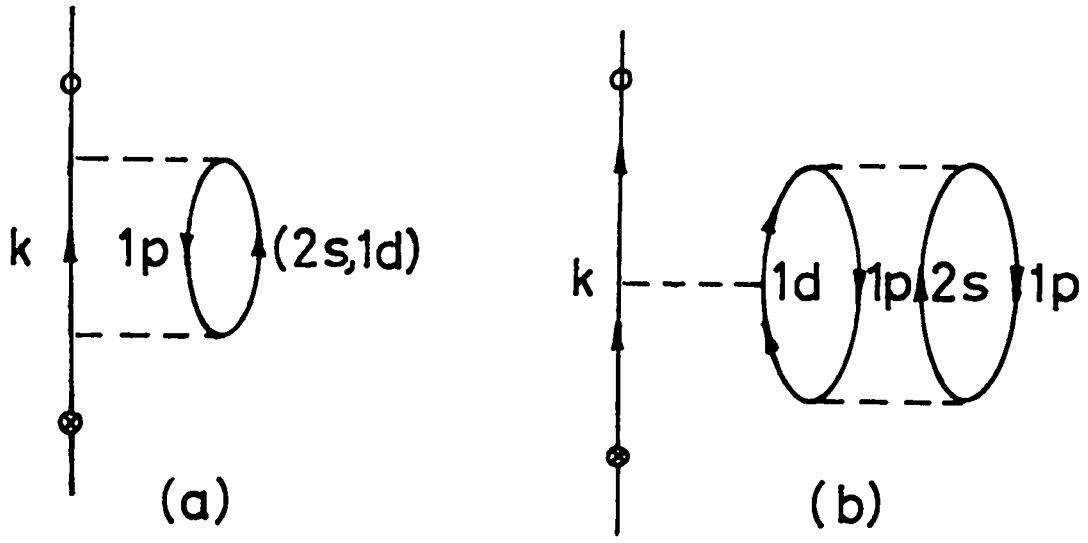

Fig. 2 Potential interactions leading to charge exchange in the two-state model. One of the interacting nucleons, "0" and "1", in particular the bound constituent, has undergone a change of state. Such can occur through the interaction as in (a) or indirectly as in (b). Direct charge exchange occurs in first order.

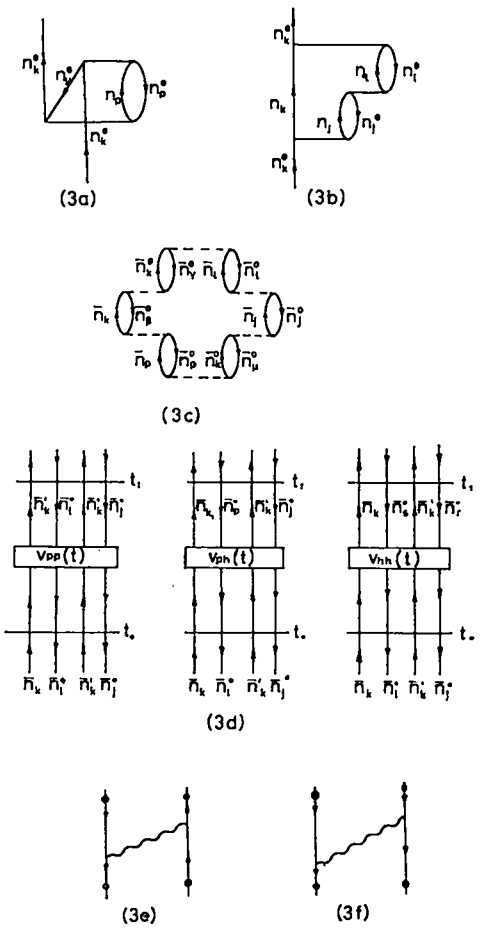

Fig. 3 Diagrammatic representations leading to charge exchange through the intermediary of two quasi-particle-single phonon vertices. The 3-particle, 2-hole interaction (a), and the 2-particle, 1-hole interaction (b) are described in terms of the A-particle ground state as physical vacuum. The interactions occur through t- or K-matrices, whichever is appropriate. The same interactions can be viewed in a representation where the ground state A+1 system defines the vacuum. Here, 2-hole, 2-particle excitations occur, symbolically represented as in (c). All of the interactions are not drawn in. The effect of these upon a given unperturbed configuration is shown in (d). Diagonalization of these residual interactions leads to the Feynman diagrams (e), for the A-particle vacuum, and (f) for the A+1 vacuum. In both of these the time-orderings are specified for the complete or single-particle propagators; ∿∿ denotes the phonon connecting ground and excited states of the A+1 system.

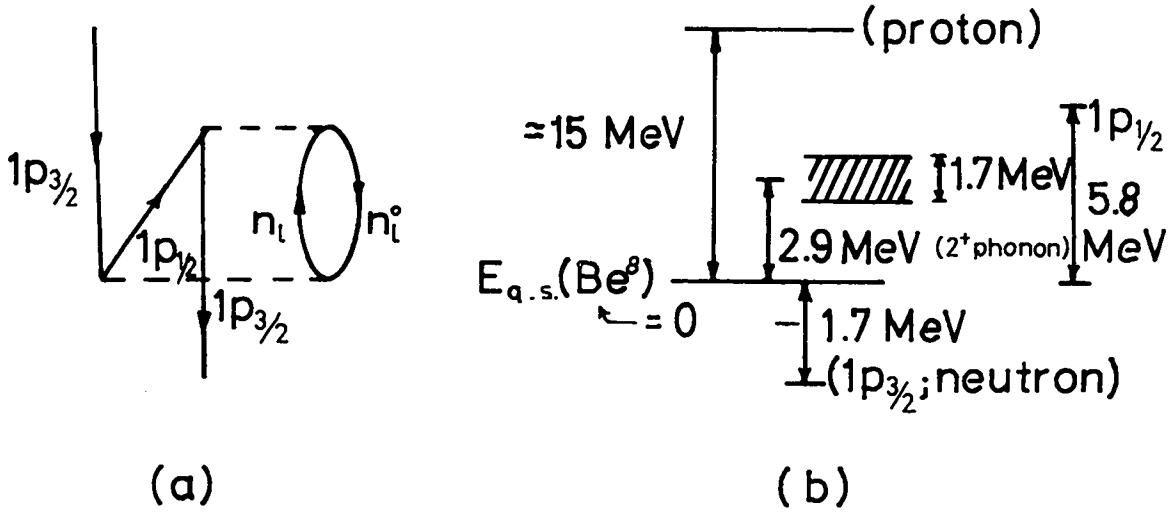

Fig. 4 Suppression of vacuum fluctuations associated with Be^8 core by $1p_{3/2}$ neutron in Be^9, (a). The corresponding process, Be^9 suppression by the added proton, has been accounted for in Fig. (3a). The location in energy of the two particle states and the phonon state is shown in (b).

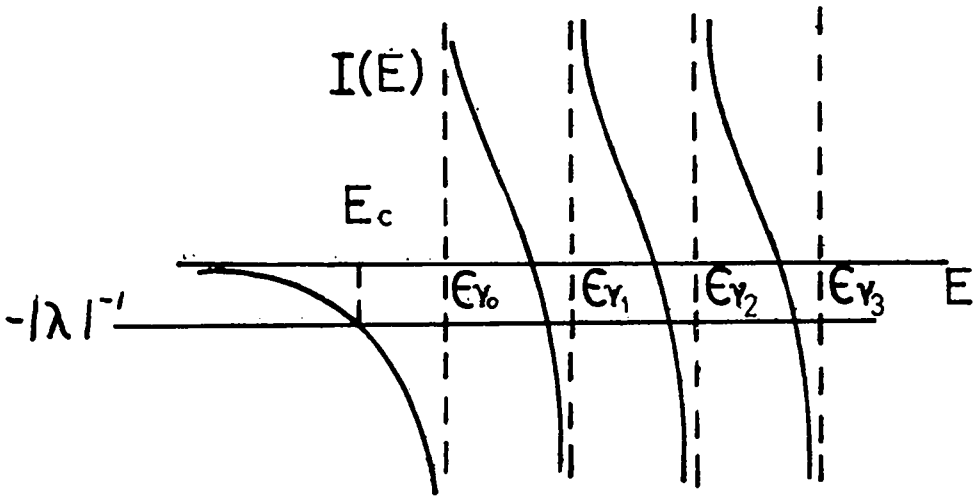

Fig. 5 Canonical plot depicting one state E_c splitting off from the spectrum ϵ_{γ_i} of independent particle excitations. The ϵ_{γ_i} refer, here, to independent pairs of particles. The lowest such excitation is ϵ_{γ_0}. The state E_c has split off because of the degeneracy or near degeneracy of the ϵ_{γ_i}, within some energy interval ω, about an energy E^*.

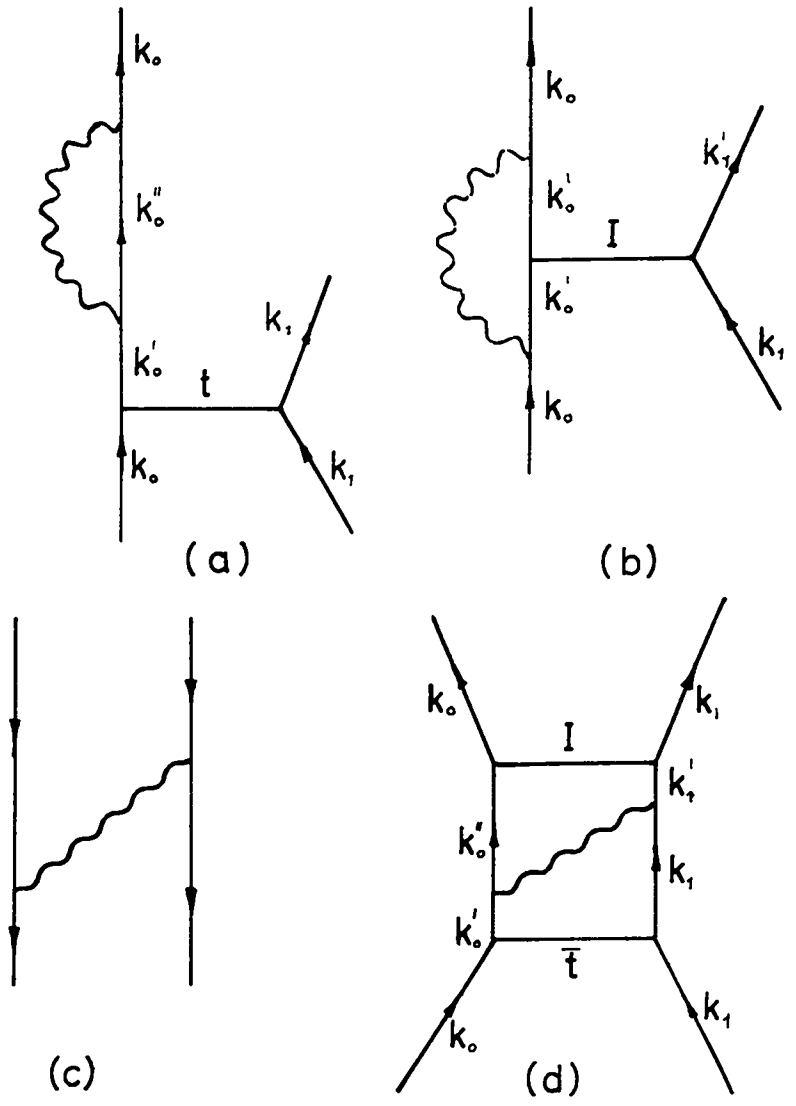

Fig. 6 Diagrammatic representations, not of the Feynman type, displaying some of the interactions involving the virtual 2+ phonon (~~~). In (a) the emission condition for the phonon is reflected by the labelling on the line for added nucleon. Apart from this self-energy insertion, there appear vertex corrections similar to (b). Two nucleons in states below the Fermi sea, core states, can exchange a vibrational phonon as in (c). This long range force is available to particles in states above the Fermi sea only through more complicated processes, such as (d). The interaction notation is $t = \bar{t} + I$.

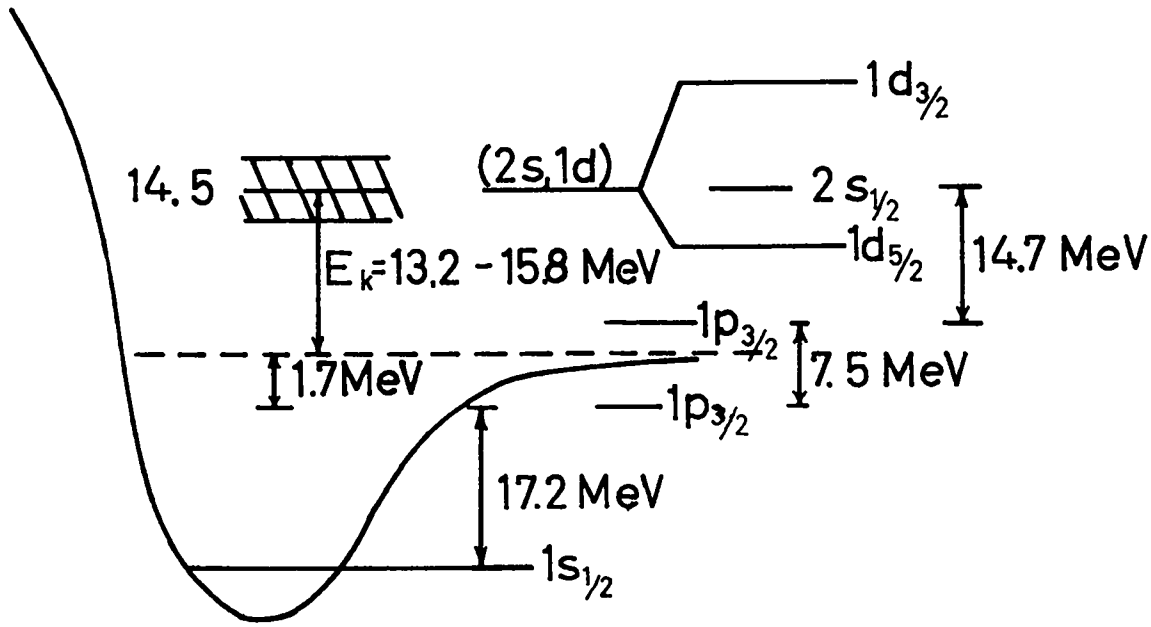

Fig. 7 Finite well representation, not to scale, of levels appearing in two-state model, (L-S) and (j-j). An oscillator representation has been for the target system. Use $\hbar\omega$ = 19.7 Mev to locate 1p, the center-of-gravity of the p-states. The added nucleon has the spectrum of states E_k.

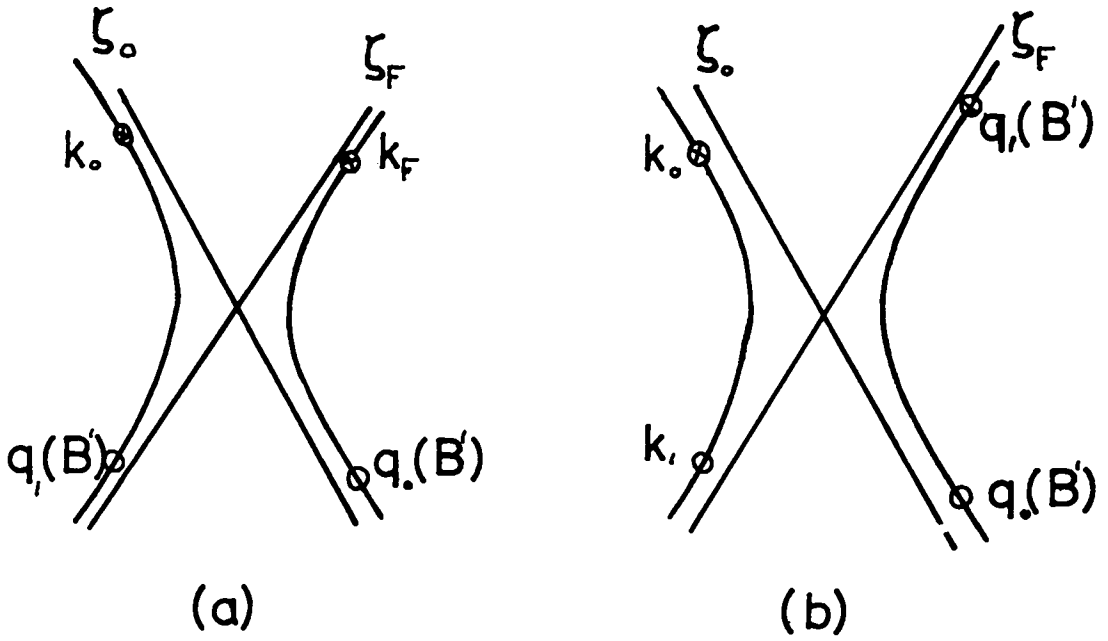

Fig. 8 Collision schematics for two t-matrix representations of charge exchange: (a) refers to Eq. (31) and implies an exchange of all of the coordinates for the two nucleons involved, hence an exchange of the quantum states; (b) refers to Eq. (34) and implies a transfer of charge between the two nucleons, no change of state occurring. The two processes are physically indistinguishable. The collision asymptotes are labelled by $\mathcal{f}_0$ and $\mathcal{f}_F$, the initial and final relative momenta.

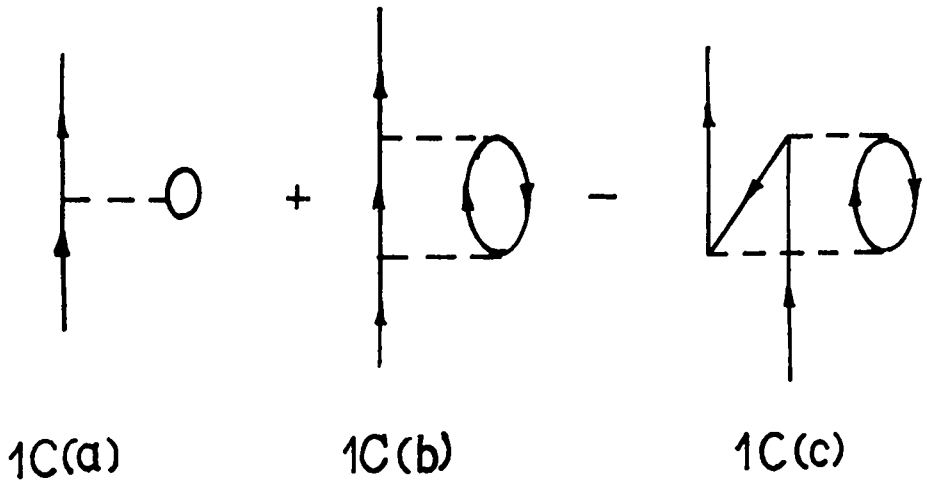

Fig. 1C Contributions to the interaction energy for the neutron incident upon the A = 9 ground state: (a) H-F potential; (b) self-energy term; (c) vacuum suppression term. These terms are those through second order in perturbation theory.

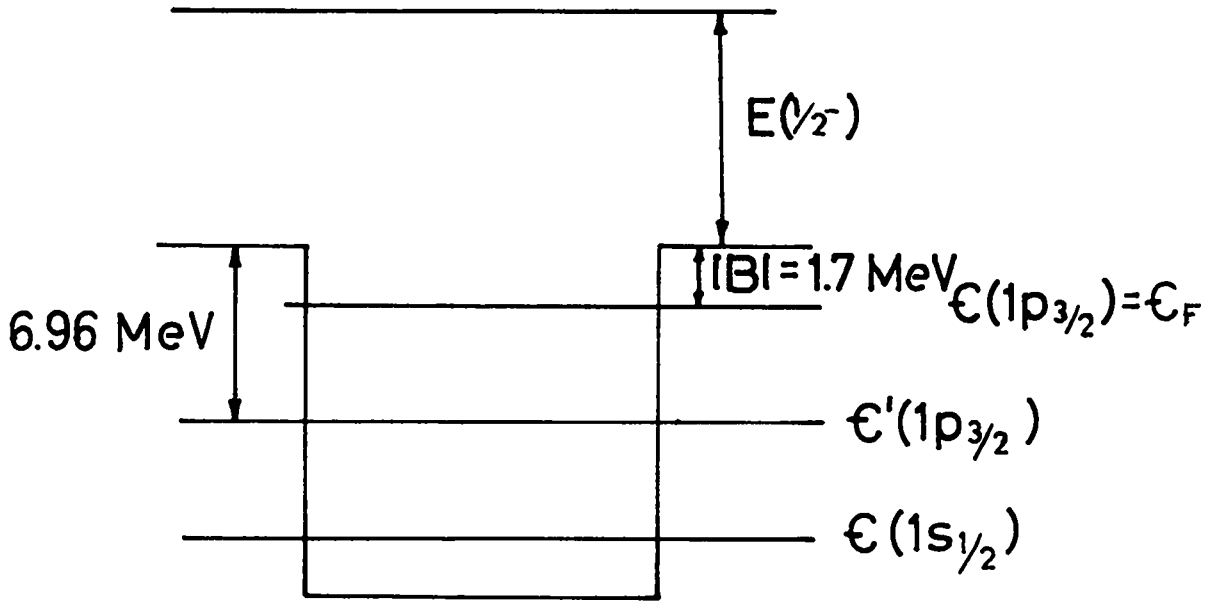

Fig. 2C Finite well applying n-Be^9 scattering; E(1/2⁻) is taken as the k.e. of incident neutron. The last neutron in Be^9 is in the state $\epsilon(1p_{3/2})$. The remaining p-shell nucleons are in the state $\epsilon'(1p_{3/2})$. A square well has been used for simplicity of presentation.